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ABSTRACT

This paper discusses the role of the culture in secondary mathematics education. A historical and philosophical look at the relationship between the real world and mathematics and mathematics education is developed. The U.S. secondary mathematics curriculum is explored both historically and philosophically to try to ascertain its changing attitudes toward applications and the culture over time. Two University of Chicago School Mathematics Project (UCSMP) texts, "Transition Mathematics" and "UCSMP Algebra" were compared with other commercial textbooks to determine the number and type of cultural contexts. The four commercial textbooks studied were: "Heath Pre-Algebra"; Merrill's "Mathematics 7"; Dolciani's "Algebra 1"; and Saxon's "Algebra 1." Among the findings, the study concluded that UCSMP materials have about three times as many cultural contexts than the four commercial texts. UCSMP materials also had many more contexts in the non-traditional applications of mathematics such as demographics, business, politics, law, technology, etc. The appendices, which constitute over half of the document, provide lists of the cultural contexts found in each of the six textbooks analyzed. The bibliography contains 50 sources. (KR)

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Culture, Literacy, and Secondary Mathematics Education

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Table of Contents

Introduction 3

A History of Mathematics and its Applications 5

Mathematics and its Applications in the U.S. Secondary Curriculum 11

The University of Chicago School Mathematics Project 21

A Textbook Comparison Study 29

Discussion 36

Conclusion: Ideas for Further Study 40

Appendix A: Cultural Contexts in *Heath Pre-Algebra* 41

Appendix B: Cultural Contexts in Merrill's *Mathematics 7* 48

Appendix C: Cultural Contexts in *Transition Mathematics* 54

Appendix D: Cultural Contexts in Dolciani's *Algebra 1* 72

Appendix E: Cultural Contexts in Saxon's *Algebra 1* 81

Appendix F: Cultural Contexts in *UCSMP Algebra* 84

Bibliography 104

Introduction

Mathematics instruction is regarded by the public in the United States with much ambivalence. There is the widespread view that someone who is proficient in mathematics is smart (regardless of the performance in other areas), and that that person has a better chance at a good job as an adult. There is evidence to support this latter belief. The August 21, 1989, issue of *USA Today*, cited a study by Estelle James, professor of economics at the State University of New York at Stony Brook, which concluded that people who enroll in more mathematics courses and receive good grades in them (A's or B's) increase their future earnings. On the other hand, mathematics seems to be the only school subject in which some people take pride in poor performance. It is culturally acceptable to say that you were never any good at mathematics; whereas illiteracy or unawareness of current events carries with it a social stigma. It has been suggested that people feel better about their success if they have overcome a barrier such as poor performance in school mathematics.

Secondary mathematics instruction is also viewed with ambivalence by mathematicians and mathematics educators. While there is great pride and devotion towards mathematics itself, there has always been dissatisfaction with the teaching of mathematics in the secondary school. There have been reform movements in mathematics education since the end of the last century, and this continues to the present.

The most recent reform documents have viewed the current secondary school mathematics curriculum as divorced from the real world. The curriculum is seen as too focussed on abstract skills and symbol manipulation, and it is unrelated to how people use mathematics in their careers and other aspects of their daily lives. While this current reform movement has its roots in reports written in 1975, two reports released this year continue in this same vein. Both of these reports were publicized in the popular media as well as being widely circulated in the mathematics education community.

The first of these reports, *Everybody Counts*, was written by the National Research Council which consisted of the Mathematical Sciences Education Board, the Board on Mathematical Sciences and the Committee on the Mathematical Sciences in the Year 2000. In it, the National Research Council outlined the strong role mathematics has in the culture of the United States.

> To function in today's society, mathematical literacy--what the British call "numeracy"-- is as essential as verbal literacy. These two kinds of literacy, although different, are not unrelated. Without the ability to read and understand, no one can become mathematically literate. Increasingly, the reverse is also true: without the ability to understand basic mathematical ideas, one cannot fully comprehend modern writing such as that which appears in the daily newspapers.
>
> Numeracy requires more than just familiarity with numbers. To cope confidently with the demands of today's society, one must be able to grasp the implications of many mathematical concepts--for example, chance, logic, and graphs--that permeate daily news and routine decisions.
>
> Literacy is a moving target, increasing in level with the rising technological demands of society. Indeed, there is some evidence that the decline in reading comprehension scores over the last several decades is due in part to the growing mathematical content of what one is required to read. It is not just computer manuals or financial reports that require an understanding of mathematical ideas; so do reports of political polls, debates about AIDS testing, and arguments over the federal deficit. Even Supreme Court decisions resemble mathematical arguments whose subject matter is law rather than numbers; often, legal cases rest as much on probabilistic inferences (for example DNA fingerprinting, fiber analysis) as on direct evidence.
>
> Functional literacy in all of its manifestations--verbal, mathematical, scientific, and cultural--provides a common fabric of communication indispensable for modern civilized society. (National Research Council, pp: 7-8)

A second report, the *Curriculum and Evaluation Standards for School Mathematics*, was written by the Commission on Standards for School Mathematics of the National Council of Teachers of Mathematics, the largest mathematics teacher organization in the United States. Before listing the specific standards, they write that "educational goals for students must reflect the importance of mathematical literacy." (National Council of Teachers of Mathematics, 1989, p. 5) The first of these goals is the need for students to value mathematics.

> Students should have numerous and varied experiences related to the cultural, historical, and scientific evolution of mathematics so that they can appreciate the role of mathematics in the development of our contemporary society and explore relationships among mathematics and the disciplines it serves: the physical and life sciences, the social sciences, and the humanities.
>
> Throughout the history of mathematics, practical problems and theoretical pursuits have stimulated one another to such an extent that it is impossible to

> disentangle them. Even today, as theoretical mathematics has burgeoned in its diversity and deepened in its complexity and abstraction, it has become more concrete and vital to our technologically oriented society. It is the intent of this goal--learning to value mathematics--to focus attention on the need for student awareness of the interaction between mathematics and the historical situations from which it has developed and the impact that interaction has on our culture and our lives. (National Council of Teachers of Mathematics, 1989, pp. 5-6)

Both of these groups explicitly mention the terms "culture" and "literacy." These terms have become popular since the publication of E.D. Hirsch's best-seller, *Cultural Literacy*, in the summer of 1987. Another best-seller, *Innumeracy*, by John Allen Paulos relates literacy to mathematics. The popular view is that education and mathematics education must better tie into the real world and the culture.

This paper looks at this question of the role of the culture in secondary mathematics education. This is a different perspective than most arguments take concerning the role of applications or of properties in the mathematics curriculum. A historical and philosophical look at the relationship between the real world and mathematics and mathematics education is developed. Then the United States secondary mathematics curriculum is explored both historically and philosophically to try to ascertain its changing attitudes towards applications and the culture over time.

Finally, materials developed by the University of Chicago School Mathematics Project, the largest mathematics curriculum development project in the United States in the last twenty years, offer a new approach to relating mathematics and the real world. The question of whether this approach significantly differs from other current approaches is researched in a small textbook comparison. The results of that research is then discussed within this historical and philosophical context and suggestions for assessing this approach are offered.

A History of Mathematics and its Applications

> On the face of it, the various studies, arithmetic, geography, language, botany, etc., are themselves experience–they are that of the race. They embody the

> cumulative outcome of the efforts, the strivings, and the successes of the human race generation after generation. They present this, not as a mere accumulation, not as a miscellaneous heap of separate bits of experience, but in some organized and systematized way–that is, as reflectively formulated. (Dewey, p. 12)

One does not say that mathematics was discovered or invented, rather it is said that mathematics developed side by side with human communication. There was an early need to describe collections of objects, whether they be body parts or rocks or groups of people; hence the need for counting numbers. Further development came with attempts to more completely describe the world:

> The primary motivation for the development of mathematics proper and the primary reason for the great importance of the subject is its value in the study of nature. Mathematical concepts and mathematical methods of obtaining knowledge have been most effective in representing and investigating the motions of the heavenly bodies and the motions of objects on and near the surface of the earth, the phenomena of sound, light, heat, electricity, and electromagnetic waves, the structure of matter, the chemical reactions of various substances, the structure of the eye, ear, and other organs of the human body, and dozens of other major scientific phenomena. (Kline, p. 7)

Although the modern base ten numeration system did not appear until around 700 A.D., and the basic arithmetic symbols such as + and ÷ did not appear until around 1500 A.D. and 1650 A.D. respectively; the processes of arithmetic as well as the study of the basic geometric shapes were known by the ancient Egyptians, Babylonians, and Chinese. This paper, however, will concentrate on mathematical developments in western civilization.

It was not until the Greeks that these processes and shapes were truly organized. They are credited for first developing mathematical systems. Thales (624-548 B.C.) is claimed to be the first true mathematician. He is called the originator of the deductive system. Pythagoras (580-500 B.C.) is credited for advancing and popularizing mathematics. Plato (427-348 B.C.) and Aristotle (384-322 B.C.) made geometry the model of a logical system. Euclid (330-270 B.C.) categorized and organized this system in his great work *The Elements*. This system was unchallenged for over 2000 years.

Thus it was in the Grecian civilization that a tension in mathematics between its use for real world situations and its beauty as a system began. *Logistica* is the translation of the term used by the Greeks for the manipulations of arithmetic used in commerce, trade, calendar reckoning and

military problems. *Arithmetica* is the translation of the term used to describe the study of the concept of number, its principles and further applications to nature. It was clear that in ancient Greece, the use of applied mathematics was seen as inferior to the study of mathematical theory. This view was expounded with most conviction and influence by the great philosopher Plato. Plato often regarded geometry as pure truth, a system where the axioms were accepted as true, and where theorems could be proved; where truth was a certainty:

> Plato regarded logistica as appropriate for the businessman and for the man of war, who "must learn the art of numbers or he will not know how to array his troops." The philosopher, on the other hand, must be an arithmetician "because he has to arise out of the sea of change and lay hold of true being." Moreover, Plato says in the *Republic*, "arithmetic has a very great and elevating effect, compelling the mind to reason about abstract number." . . . As in arithmetic Plato saw a gulf separating the theoretical and computational aspects, so also in geometry he espoused the cause of pure mathematics as against the materialistic views of the artisan or technician. (Boyer pp. 95-96)

Mathematics was to be admired on its own merits, not because it helped to solve real world problems.

Using this philosophy, the Greeks mostly expanded the field of geometry, but they did considerable work in arithmetic and number theory as well. These were terrific accomplishments especially considering that they did not have the base ten numeration system, nor did they have the coordinate plane. Amazing advancements were made in geometry. They developed the method and theory of constructions, where the only allowable tools were a straightedge and compass: "Plato may . . . have been largely responsible for the prevalent restriction in Greek geometrical constructions to those that can effected by straightedge and compasses alone." (Boyer, p. 96) They developed many theorems based on only five axioms and five postulates from Euclid. In fact, it was not until the 1800s that mathematicians again turned in force to the field of geometry.

It is certain that *The Elements* of Euclid is the most powerful mathematics textbook ever. It is claimed that it has been through more printings of any book except the Bible. It covers the fundamentals of geometry, number theory, and irrational numbers then called incommensurables:

> Evidently Euclid did not stress the practical aspects of his subject, for there is a tale told of him that when one of his students asked of what use was the study

> of geometry, Euclid asked his slave to give the student threepence, "since he must needs make gain of what he learns." (Boyer, p. 111)

There were other textbooks before *The Elements*, but none has survived. What this means is that the only ancient textbook and the most influential textbook of all time stressed the abstract in mathematics. If there is a historical cause to the lack of applications in school mathematics, surely the role of *The Elements* plays a big part.

Ironically, the greatest Greek mathematician, and arguably the greatest mathematician of all time, worked with many applications of mathematics and did not just work with the theory of geometry. Archimedes (287-212 B.C.) is best known for his quote, "give me a place to stand on and I will move the earth." He seemed to be the first person who was fluidly able to apply mathematics to new problems; as well as discover new mathematics emanating from applications.

As Bell writes in *Men of Mathematics*:

> Leaving aside his great contributions to astronomy and mechanical invention we shall give a bare and inadequate summary of the principal additions which Archimedes made to pure and applied mathematics.
>
> He invented general methods for finding the areas of curvilinear plane figures and volumes bounded by curved surfaces, and applied these methods to many special instances, including the circle, sphere, any segment of a parabola, the area enclosed between two radii and two successive whorls of a spiral, segments of spheres, and segments of surfaces generated by the revolution of rectangles (cylinders), triangles (cones), parabolas (paraboloids), hyperbolas (hyperboloids), and ellipses (spheroids) about their principal axes. He gave a method for calculating π (the ratio of the circumference of a circle to its diameter), and fixed π as lying between 3 1/7 and 3 10/71; he also gave methods for approximating to square roots which show that he anticipated the invention by the Hindus of what amount to periodic continued fractions. In arithmetic, far surpassing the incapacity of the unscientific Greek method of symbolizing numbers to write, or even to describe, large numbers, he invented a system of numeration capable of handling numbers as large as desired. In mechanics he laid down some of the fundamental postulates, discovered the laws of levers, and applied his mechanical principles (of levers) to calculate the areas and centers of gravity of several flat surfaces and solids of various shapes. He created the whole science of hydrostatics and applied it to find the positions of rest and of equilibrium of floating bodies of several kinds. (Bell, pp. 30-31)

Archimedes lived at a time of a transition to a more efficient Ionian numeration, and he used this to make his many contributions to logistica. He helped solve unsolved problems in geometry, and he was not reluctant to move away from the rigid straightedge and compass constructions. He wrote many treatises of his results, many of which are believed to be lost, but those that remain

include *On the Equilibrium of Planes*, on the lever and other physics; *On Floating Bodies*, on water displacement and other fluid properties; *Psammites* ("Sand-Reckoner"), on large numbers (greater than the number of grains of sand); *On the Measurement of the Circle*, his most popular work in the Medieval period, *On Spirals*, his most difficult work which anticipates rectangular and polar coordinates, *Quadrature of the Parabola*, which anticipates integral calculus, and *On the Sphere and Cylinder*, believed to be Archimedes's favorite work. This is only a partial list.

It is now known that Archimedes would begin many of his studies with mechanical, real world problems. In his work *The Method*, lost until 1906, he outlined many of his investigations. He would start with a real world problem, and then figure out a mathematical model to describe it. He then would look at the model and see how it could further contribute to mathematics.

It is necessary to discuss Archimedes because he stood in such stark contrast to the other Greek mathematicians, not only for his brilliance, but for his attitude towards mathematics. He was not entrapped in this dichotomy of applied versus theoretical mathematics. This shows that even in ancient times, there need not have been the assumption that theoretical mathematics was somehow better or more pure. Pure theory was just one facet of mathematics. The lack of applications in secondary mathematics education today derives from the fact that Euclid's and Plato's view of mathematics took hold over the view expressed by Archimedes's life work.

In western civilization, the history of the teaching of geometry has been patterned after Euclid. In the middle ages *The Elements* was the mathematics textbook. This was true even as of colonial times. Currently it is still the model for United States geometry textbooks. The emphasis is on teaching a strict deductive system from a set of postulates and doing proof. In geometry it has been traditional to teach a deductive, abstract system without much, if any regard, for the real world.

There is a kicker to this story, and that is that the geometry of *The Elements* does not accurately describe the real world. In 1829, Nicolai Ivanovitch Lobachevsky published the article "On the Principles of Geometry" in *Kazan Messenger*. This is the first published account of non-Euclidean geometry. It is now recognized that Karl Fredrich Gauss had done some private work

on non-Euclidean geometry five years earlier, and that at about the same time Janos Bolyai was discovering a non-Euclidean geometry. Non-Euclidean geometry became more acceptable through the works of G. F. B. Riemann, and it was then that the historic contributions of Lobachevsky were truly credited.

It was Riemannian geometry that described the surface of a sphere, which describes the earth (although the earth is not a perfect sphere). In Euclid's geometry, the sum of the angles of a triangle is proved to be 180°. On the surface of the earth, if a triangle is formed by the North Pole and two points on the Equator, the angle measures of this triangle will sum to more than 180°.

The ramifications of non-Euclidean geometry on mathematical philosophy cannot be understated. Before non-Euclidean geometry, the five geometric postulates in *The Elements* were viewed as "self-evident truths." These postulates were absolutely true, and the world was based on them. Now it is recognized that postulates are assumptions. In non-Euclidean geometry it was shown that a different set of postulates could lead to a consistent and useful geometry completely different to that of Euclid. This was the end of geometry or mathematics as the absolute truth.

As a side, the notion that mathematics was not the absolute truth was a hard one to accept. Bertrand Russell tried to create a rigorous logical system starting with proving everything using the accepted postulates of logic. He failed in this effort, as he had no chance of success. This was shown in the next great shake-up of mathematical philosophy when Kurt Gödel showed that any logical system of any power had to be either incomplete or contain inconsistencies.

Plato's most likely reason for elevating arithmetica over logistica was that arithmetica was philosophically better, it described truth. In the dialogue *Meno*, Socrates uses mathematics as a model that philosophy should aspire to. His dialogues such as *Gorgias* were all designed to make his "opponents" prove their philosophic claims. To Plato, the abstraction of mathematics had this wonderful quality that its tenets were proven, they were much more philosophical and nearer the philosophical ideal of absolute truth, than a rudimentary application was.

The philosophic tables have been completely turned around in current times. It is certain to our twentieth century minds that there is no absolute truth; nor is there an absolute mathematical or

logical system. There exists a myriad of mathematical systems to play with theoretically or to apply to a real world problem. It would seem that the view of Archimedes would be prevalent. For a given problem, it is appropriate to use or discover the correct mathematics to model it. For some given mathematics, find problems that it applies to or deduce some further mathematics.

Still, 160 years after the birth of non-Euclidean geometry, the influence of Plato and Euclid lingers.

> The ideal mathematician's work is intelligible only to a small group of specialists, numbering a few dozen or at most a few hundred. This group has existed only for a few decades, and there is every possibility that it may become extinct in another few decades. However, the mathematician regards his work as part of the very structure of the world, containing truths which are valid forever, from the beginning of time, even in the most remote corner of the universe. (Davis and Hersh, p. 34)

While this portrait exaggerates the situation, there are many mathematicians who feel this way.

Mathematics educators, however, must deal with the public and the children of the United States. It is worthy to note what curricular choices have been made concerning mathematics and the real world over time. The next section historically explores the changing views concerning the secondary curriculum.

Mathematics and its Applications in the U.S. Secondary Curriculum

The history of mathematics education in this country offers some insights into the role of mathematics vis-a-vis the real world. Mathematics education began in this country with the teaching of arithmetic at the university level in 1680. The arithmetic of colonial times was summarized by Florian Cajori's discussion of Dilworth's School-Master's Assistant, the most noted text at the time: "The whole book is nothing but a Pandora's box of disconnected rules. It appeals to the memory exclusively and completely ignores the mind of the reader." (Cajori, p. 16) He singles out this particular text since it was typical of those that originated in England: "All arithmetics of that time were much alike. The criticisms upon one will therefore apply to all."

(Cajori, p. 17) Thus colonial arithmetic was not an abstract course, nor a particularly practical course, but one which exercised mental discipline and would "work out" the mind. Thus began a tradition, which in some sense carries on to this day. Certainly for many elementary students today, the drill of pencil and paper algorithms has neither much practicality nor offers any insight into the structure of mathematics, it is a subject which tries the discipline of many a student.

During the entire nineteenth century, there underwent the process of developing an educational system for the youth of the United States. In school mathematics, the changes prior to 1890 consisted of bringing arithmetic into the elementary school and algebra and geometry into the secondary school. Trigonometry was still prevalent at the university level, but it was ready to move into the secondary school curriculum around the turn of the century. Harvard required algebra for admission beginning in 1820, while Yale and Princeton required it in 1847 and 1848 respectively. Yale was the first university to require geometry, starting in 1865; Princeton, Michigan and Cornell followed three years later.

The field of secondary mathematics education has its roots in the very late 1800s and it wasn't until this century that a focus upon what mathematics should be taught to all children was addressed. A good starting point for focussing on the historical treatment of school mathematics and its relationship to the real world in the modern secondary curriculum is to start with the situation as it was in 1890:

> Textbooks in the three subject areas of school mathematics (arithmetic, algebra, and geometry) provide evidence that the operant philosophy in determining curricular content was mental discipline. Little attention was given to the use of mathematics as a tool. Indeed, mathematics was learned primarily for its own sake and for the purpose of training the faculties of the mind. Subjects were compartmentalized, and within a given subject little or no effort was made to establish interrelationships of ideas. The concern for logical organization took precedence over the immediate utility of mathematics. Little effort had been made to adjust college-oriented mathematics to less mature students. (Osborne and Crosswhite, pp. 156-157)

In 1890, the National Council of Education (part of the National Education Association) appointed a committee to address the public's dissatisfaction with secondary education. This committee suggested in 1892 that there was a need for a huge study looking at the structure of the

secondary school curriculum. The NEA then appointed the Committee of Ten on the Secondary School Studies to look into nine different curricular areas. The subcommittee of ten people looking into mathematics was called the Conference of Mathematics.

In its conclusions, the Conference of Mathematics suggested that it was more important for students to have the mental discipline of mathematics than the work or knowledge of its applications. They wrote:

> . . . the conference recommends that the course in arithmetic be at the same time abridged and enriched; abridged by omitting entirely those subjects which perplex and exhaust the pupil without affording any really valuable mental discipline, and enriched by a greater number of exercises in simple calculation and in the solution of concrete problems.
>
> Among the subjects which should be curtailed, or entirely omitted, are compound proportion, cube root, abstract mensuration, obsolete denominate quantities, and the greater part of commercial arithmetic. Percentage should be rigidly reduced to the needs of actual life. In such subjects as profit and loss, bank discount, and simple and compound interest, examples not easily made intelligible to the pupil should be omitted. Such complications as result from fractional periods of time in compound interest are useless and undesirable. (National Education Association, p. 105)

For college-bound students, the Conference of Mathematics suggested that an emphasis on the theoretical, systematic aspects of mathematics be made. In algebra it was important that rules be developed inductively, and in geometry it was recommended that the level of rigor be raised in geometric proof. In a somewhat different light wrote that "in the case of those who do not intend to go to college, but to pursue a business career, the remainder of the term which has been allotted to algebra might well be devoted to book-keeping, and the technical parts of commercial arithmetic." (National Education Association, p. 107) This means that a semester's worth of some applications was all that was needed for those less proficient students.

Both the Committee of Ten and the Conference of Mathematics were convinced that the current approach of simply laying down rules was unsound, but their recommendations favored developing the theory and abstractions of mathematics much more than concentrating on its real world uses.

There appeared to be the recognition in the early part of the twentieth century that practical problems provided excellent motivation for students. This force for practicality gained strength

and complexity outside the United States as well as inside it. In 1912, some twenty years after the Committee of Ten Report, the International Commission on the Teaching of Mathematics noted that utilitarian aims of mathematics were becoming increasingly more important. In their writings on the school mathematics curriculum, mathematicians and mathematics educators Felix Klein, John Perry, and Eliakim Hastings Moore incorporated pleas for this utilitarianism.

Concentration of issues solely pertaining to the secondary curriculum may have its start with the formation of the National Committee on Mathematical Requirements under the auspices of the Mathematical Association of America (MAA) "for the purpose of giving national expression to the teaching of mathematics, which had gained considerable headway in various parts of the country, but which lacked the power that coördination and united effort alone could give." (Young, p. vii)

This report is commonly known as the 1923 Report, and the philosophy behind it was stated clearly:

> The primary purposes of the teaching of mathematics should be to develop those powers of understanding and of analyzing relations of quantity and of space which are necessary to an insight into and control over our environment and to an appreciation of the progress of civilization in its various aspects, and to develop those habits of thought and of action which will make these powers effective in the life of the individual.
>
> All topics, processes, and drill in technique which do not directly contribute to the development of the powers mentioned should be eliminated from the curriculum. . . .
>
> Drill in algebraic manipulation should be limited to those processes and to the degree of complexity required for a thorough understanding of principles and for probable applications either in common life or in subsequent courses which a substantial proportion of the pupils will take. It must be conceived throughout as a means to an end, not as an end in itself. (Young, pp. 13-15)

The one element that was quite unique to the 1923 Report was contained in its pronouncement that the aims of mathematical instruction were of three types: practical, disciplinary, and cultural. There had not been curricular discussion of the role of mathematics in the culture prior to this. The report stated,

> By cultural aims we mean those somewhat less tangible but none the less real and important intellectual, ethical, esthetic or spiritual aims that are involved in the development of appreciation and insight and the formation of ideals of perfection. . . .
>
> More specifically we may mention the development or acquisition of
> (1) *Appreciation of beauty* in the geometrical forms of nature, art, and industry.
> (2) *Ideals of perfection* as to logical structure, precision of statement and of thought, logical reasoning (as exemplified in the geometric demonstration), discrimination between the true and false, etc.
> (3) *Appreciation of the power of mathematics*–of what Byron expressively called "the power of thought, the magic of the mind"–and the rôle that mathematics and abstract thinking, in general, have played in the development of civilization: in particular in science, in industry, and in philosophy. (Young, pp. 12-13)

Thus the cultural aims of the 1923 Report are quite different from what "cultural literacy" is today. The cultural aims appear to apply here to the culture of mathematics. The three points in the preceding quote are three common reasons of why pure mathematicians enjoy mathematics. In fact, the term "cultural" here seems a way to make abstract mathematics more acceptable in school mathematics. The link between mathematics and the real world could thus only occur in the practical strand. (The disciplinary strand meant that drill and practice still was seen as a sound way to teach.)

The report's aims, however, were broad and, at times, vague. Its recommendations for content in specific courses included mastery of arithmetic algorithms, abstractions of arithmetic for algebra, and a long list of geometric theorems and constructions. By the time the curricular content was outlined, the practical applications were lost in the shuffle. Thus both teachers and textbook authors (or publishers) could find something in the report which justified whatever they were doing or wished to do.

"The evolution of mathematics education from 1920 to the post-World War II years hinged primarily on one issue, namely, the usefulness of mathematics." (Osborne and Crosswhite, p. 234) It seemed to be a period of failed attempts at reforming mathematics education, but greater scrutiny leads to the conclusion that it was a period perhaps more of aborted attempts at reform than failed ones. Critics felt that the influence of the 1923 Report was too slow to be felt in the

classroom. It was not then realized that changing the curriculum takes time. There was, in fact, a variety of methods employed in the mathematics classrooms across the country. It was possible to find methods of teaching mathematics as diverse as teaching pure drill and proof, teaching arithmetic through measurement, putting consumer mathematics in the junior high school, putting business applications in the curriculum, using a psychological approach, and using military applications. The overall feeling though was, that by World War II, mathematics education was still in strong need of reform. The testing of World War II recruits confirmed this.

In the 1950s, United States was becoming a society increasingly reliant on the scientific and technological abilities of its government and industry. This, of course, meant that it was necessary to educate people who would be comfortable and proficient in mathematics and the sciences. In October, 1957, the Soviet launching of the satellite Sputnik dramatically confirmed this view and convinced the public of the need for strong mathematics education. Although this event brought in an influx of new money for curricular reform projects, the actual process of reforming the secondary mathematics curriculum had begun in the early 1950s.

In 1951 the University of Illinois Committee on School Mathematics (UICSM) was formed to study both the content and pedagogy of secondary school mathematics. UICSM has certainly had its influence: it has served as an administrative model of a curricular project, having received support through both the government and private foundations, and its conclusions marked the beginning of what is commonly termed the "new math."

Max Beberman, professor at the University of Illinois, directed the project and in the Inglis Lecture in 1958, he stated the program and philosophy of UICSM:

> The curriculum developer . . . has in mind the traditional expectations of what a college-bound high school graduate should know of mathematics. He is expected, among other things, to solve equations (singly and in systems), to use algorisms and formulas in transforming algebraic and trigonometric expressions, to deduce theorems from postulates, to find ordered pairs which belong to relations, to graph relations, and to apply his knowledge to a host of geometric and physical problems.

> We think that these objectives are sensible and proper, and our curriculum is planned accordingly. But we insist upon an important qualification: the student must *understand* his mathematics. Now the word "understand" and its close relative "meaningful" have been bandied about in education circles to a point where just about everyone pledges allegiance to the goal of teaching meaningful and understandable mathematics. We have tried to translate these words into operational terms. We believe that a student will come to understand mathematics when his textbook and teacher use unambiguous language and when he is enabled to discover generalizations by himself. (Osborne and Crosswhite, pp. 253-254)

The materials created by UICSM promoted the abstractions inherent in mathematics. Their underlying assumption appeared to be that if students were competent in the mathematical structure and in explaining the underlying principles in how equation solving and algorithm creation were performed, then there would certainly be no problem with students applying the mathematics. It was reminiscent of Plato believing that arithmetica was mathematics, and logistica was not as important.

This philosophy would not have impacted upon the secondary mathematics curriculum had there not been other commissions, projects, and professional groups supporting this view. This in itself did not guarantee acceptance, but since the community was ready for change, the UICSM philosophy became accepted.

In 1955, the Commission of Mathematics of the College Entrance Examination Board was formed to study the mathematical needs of the student population. It summarized its major proposals in a nine-point program for college-capable students.

1. Strong preparations, *both* in concepts *and* in skills, for college mathematics at the level of calculus and analytic geometry.
2. Understanding of the nature and role of deductive reasoning–in algebra, as well as in geometry.
3. Appreciation of mathematical structure ("patterns")–for example, properties of natural, rational, real, and complex numbers.
4. Judicious use of unifying ideas–sets, variables, functions and relations.
5. Treatment of inequalities along with equations.
6. Incorporation with plane geometry of some coordinate geometry, and essentials of solid geometry and space perception.
7. Introduction in grade 11 of fundamental trigonometry–centered on coordinates, vectors, and complex numbers.

8. Emphasis in grade 12 on elementary functions (polynomial, exponential, circular).

9. Recommendation of additional alternative units for grade 12: *either* introductory probability with statistical applications *or* an introduction to modern algebra. (College Entrance Examination Board, pp. 33-34)

These points do not mention any uses or applications except that introductory probability with statistical applications might be a nice alternative unit for grade 12. The Commission went into detail on these recommendations and it is clear that it felt that the theory and processes of mathematics should be taught and not its uses.

The National Council of Teachers of Mathematics (NCTM) provided support for this abstraction of school mathematics through its publications. The *Mathematics Teacher*, its monthly journal for secondary school teachers provided a forum for the Commission as well as other reformers. Its yearbooks *Insights into Modern Mathematics* and *The Growth of Mathematical Ideas, Grade K-12* both lent support for curricular change. Many small curricular projects were begun in the 1950s and together they proved to be a boon to the reform movement, even though individually none made a great impact.

The most prominent of all these reform groups was the School Mathematics Study Group (SMSG) which was formed by professional mathematicians. Yale University acted as first host to the project and Edward G. Begle of its department of mathematics directed the project. Begle used the language of utility to recommend the teaching of abstract mathematics as he wrote:

> A far more fundamental reason for the improvement of mathematics education came from the realization that the society in which we live is rapidly changing. Our society is far more dependent on science and technology and, hence, on mathematics than was the case a generation ago. Mathematics is a rapidly growing subject. New mathematics is being invented at an increasing rate and finds applications in more and more different aspects of our culture. Many important mathematical skills, techniques, and ideas, which evolved only within the last generation, are now employed in science, industry, and business. We can be sure that mathematics will continue to grow, that new applications of mathematics will be found, and that a larger percentage of our citizens will need mathematics as part of the intellectual equipment for their work.

> Today we can predict neither the particular mathematical skills which will be important a generation from now nor the particular individuals among our present students who will use these skills in the future. It is, therefore, essential that we provide for all our students not only the fundamental skills which we now know to be important but also a thorough understanding of the basic mathematics which provides the foundation for these skills and will provide the foundation for the new skills of the future. Our mathematics curriculum must therefore be reformed by including materials necessary to an understanding of the basic concepts and structures of mathematics. (Begle, p. 136)

He then outlined the objectives for SMSG:

> The first of these was the provision for an improved curriculum for the schools, a curriculum which would, on the one hand, preserve the important skills and mathematical techniques which experience has shown to be important and useful, and, at the same time, provide students with a deeper understanding of the mathematics underlying these skills and techniques, since it would be this understanding of mathematics which the future will certainly require.
>
> The second objective was to provide materials and direction for the preparation of teachers to enable them to teach such an improved curriculum. . . .
>
> The third objective was that of making mathematics more interesting in order to attract more students to the subject and hold them for a longer time. (Begle, p. 140)

The materials, however, were perceived by the public as very abstract, and they were seen as counterproductive to students learning even the basic facts of mathematics. Those trained in the rigid skills of the curriculum before World War II could not appreciate this new approach.

There is some evidence, although even to this date the data has not been processed, that this public view not only exaggerated the situation but was actually mistaken.

> The new math appeared at a time when there was a national need for people trained in up-to-date science and mathematics. Consequently honors classes were established in many schools, and the Advanced Placement program became quite popular for better students. The new math was (and in some places still is) the curriculum for these students. By the early 1970s a shortage of scientists had turned into a surplus. Thought economists would probably argue that supply and demand in the job market played the largest role in turning the shortage into a surplus, the effect of the new math was to ensure that those who got jobs were of higher quality than would have otherwise been the case.
>
> Thus in some ways the new math was quite successful. Yet the public view is that this revolution was a failure. The public perception cannot be dismissed lightly, for it has made governmental agencies wary of putting money into curriculum projects, and this caution seems to have extended to the mathematics education profession as a whole. From 1972 until the time of writing this paper, not one large-scale curriculum project in secondary school mathematics has existed in North America. (Usiskin, 1985, p. 3)

The strength of the public reaction is directly attributable to the success of SMSG in distributing its materials. In the academic year 1961-1962, 5 million copies of SMSG materials

were purchased. They were the most extensively used non-commercial materials in school mathematics ever in this country. In 1963, at the height of this movement, the Cambridge Conference on School Mathematics, a group of mathematicians, wrote a report describing potential curriculum for the end of the century. Their belief was to take the goals of SMSG even further and make the curriculum more demanding, more advanced, and more abstract. The reception to this report was that it might be too ambitious and too radical.

Due to the public's strong reaction, a new trend in mathematics curriculum began. Beginning around 1974, textbook publishers came out with texts following the movement usually described by the glorious catchphrase "back to basics". These texts consisted of drill exercises upon drill exercises. For elementary school students, this meant performing algorithms, for algebra students it was pages of manipulations. These texts were quite popular and had good sales. Overall, the effect was to put mathematics education back into the situation it was in when the 1923 Report came out: the curriculum had neither applications nor abstractions.

From new math to the middle 1970s, mathematics textbooks seemed to drop all mention of the utility of mathematics. For a full generation the notion that mathematics was unrelated to the real world could not be countered by looking at textbooks. It is no surprise that there should be public sentiment which supports the view that mathematics is unrelated to the real world. It was at this time that mathematics educators had to again look at the curriculum and try to spur changes. The reports that came out from 1975 to the present will be discussed as to how they affected the development of curricular materials by the secondary component of the University of Chicago School Mathematics Project (UCSMP), currently the largest and most influential school mathematics curriculum project in the country.

The University of Chicago School Mathematics Project

The Amoco Foundation gave funding to form UCSMP in 1983 to help improve school mathematics education in the United States. The aim was to prepare middle ability students to be more mathematically literate in an increasingly technological society. The secondary component of this project is engaged in the design and production of textbooks for grades 7-12 in line with recommendations from school reform reports since 1975. The secondary component of UCSMP received additional funding in 1985 from the Carnegie Corporation of New York, in 1986 from the General Electric Foundation, and in 1989 from the Ford Motor Company.

The reports which UCSMP studied were of two types. The first type of report was written by mathematicians and mathematics educators. These were prominent in the late 1970s and served as a response to the back to basics movement. The second type of report looked at education as a whole in the United States, not just mathematics education. These reports began to appear after 1980 and reflected a broad based dissatisfaction by the public with the entire U.S. educational system. It is necessary to look at these reports to understand how UCSMP's approach and philosophy was developed towards addressing the problem of the isolation of school mathematics. One of the first of this generation's reports was done in 1975 by the National Advisory Committee of Mathematics Education (NACOME). The NACOME Report contained several recommendations, the third of which was titled Curriculum Content:

> Curriculum content, subject to the flux of accelerating change in all areas of our society, cannot be viewed as a fixed set of goals or ideas; it must be allowed to emerge, ever changing, responsive to the human and technological lessons of the past, concerns of the present, and hopes for the future. With this in mind, no definitive curriculum can ever be recommended. At benchmark 1975, the National Advisory Committee on Mathematics Education sees the following recommendations as reasonable and essential features of a contemporary mathematics curriculum
>
> a) that logical structure be maintained as a framework for the study of mathematics.
>
> b) that concrete experiences be an integral part of the acquisition of abstract ideas.

> c) that the opportunity be provided for students to apply mathematics in as wide a realm as possible–in the social and natural sciences, in consumer and career related areas, as well as in any real life problems that can be subjected to mathematical analysis. . . .
>
> h) that instructional units dealing with statistical ideas be fitted throughout the elementary and secondary school curriculum. (NACOME, pp. 138-139)

It is interesting to note that in the backlash against back to basics the report expressed desire both for applications and the usefulness of mathematics in the real world as well as mathematics as a deductive system.

In 1977, the National Council of Supervisors of Mathematics (NCSM) used the rhetoric of back to basics by publishing its "ten basic skill areas", which were very different from what the back to basics curriculum was offering. Its ten basic skills were problem solving (which included the study of heuristics needed to approach a problem); applying mathematics to everyday situations; alertness to the reasonableness of results; estimation and approximation; appropriate computational skills (which included using a calculator for most computations); geometry; measurement; reading, interpreting, and constructing tables, charts, and graphs; using mathematics to predict; and computer literacy. It is clear that this group felt that students should be acquainted much more with the usefulness of mathematics in the real world than with the intrinsic beauty or abstraction of mathematical systems.

While this report concentrated on the basics everyone should know, the NCTM and the MAA put out recommendations in mathematics education for college bound students. These recommendations were more conservative and concentrated on how students and teachers could upgrade their study and teaching habits. However, recommendation nine addressed the curriculum:

> Schools should frequently review their mathematics curriculum to see that it meets the needs of its students in preparing them for college mathematics. School districts that have not conducted a curriculum analysis recently should do so now, primarily to identify topics in the curriculum that could be either omitted or de-emphasized, if necessary, in order to provide sufficient time for the topics included in this statement. We suggest, for example, that the following could be de-emphasized or omitted from the curriculum:

(a) Logarithmic calculations that can better be handled by calculators or computers.

(b) The extensive solving triangles in trigonometry.

(c) Proofs of superfluous or trivial theorems in geometry. (NCTM–MAA)

This recommendation is noteworthy because it is the first time a report explicitly mentioned some topics which could be omitted from the curriculum. The topics that were mentioned, however, barely had a minor role, if any, in the curriculum at the time.

In 1980, the NCTM published its report, *An Agenda for Action* which stated that problem solving should be the focus of school mathematics. Within the report, NCTM gave full support to the new definition of basic skills:

- There should be increased emphasis on such activities as—
  - — locating and processing quantitative information;
  - — collecting data;
  - — organizing and presenting data;
  - — drawing inferences and predicting from data;
  - — estimating measures;
  - — measuring using appropriate tools;
  - — mentally estimating results of calculations;
  - — calculating with numbers rounded to one or two digits;
  - — using technological aids to calculate;
  - — using ratio and proportion to deal with rate problems in general and with percent problems in particular;
  - — using imagery, maps, sketches and diagrams as aids to visualizing and conceptualizing a problem;
  - — using concrete representations and puzzles that aid in improving the perception of spatial relationships.
- There should be decreased emphasis on such activities as—
  - — isolated drill with numbers apart from problem contexts;
  - — performing paper-and-pencil calculations with numbers of more than two digits;

— mastering highly specialized vocabulary not useful later either in mathematics or in daily living;

— converting measures given in one system to corresponding measures in another system;

— working with tables whose usefulness as aids to calculation has been supplanted by calculators and other technological aids (e.g., numerical computations with logarithms and cologs). (NCTM, p. 7)

The most famous of the national reports addressing the education system was *A Nation at Risk* put out by the National Commission on Excellence in Education. They did mention the school mathematics curriculum explicitly:

> The teaching of *mathematics* in high school should equip graduates to: (a) understand geometric and algebraic concepts; (b) understand elementary probability and statistics; (c) apply mathematics in everyday situations; and estimate, approximate, measure and test the accuracy of their calculations. In addition to the traditional sequence of studies available for college-bound students, new, equally demanding mathematic curricula need to be developed for those who do not plan to continue their formal education immediately. (National Commission on Excellence in Education, p. 25)

Also in 1983, the College Board Educational Equality Project published its report, *Academic Preparation for College*. It prefaced its remarks on mathematics with the following remarks:

> All people need some knowledge of mathematics to function well in today's society. Mathematics is an indispensable language of science and technology, as well as business and finance. All people, therefore, need some fluency in this language if they are to contribute to and fare well in our contemporary world.
>
> More than at any time in the past the knowledge and appreciation of mathematics is essential to students' intellectual development. The advances of recent years in computer science and other highly technical fields such as space science have opened new horizons to those trained in mathematics. Young people who avail themselves of the opportunity to gain strong preparation in mathematics and in the sciences not only will grow intellectually but also keep open the door to a wide range of career choices.
>
> Students going to college need mathematical skills beyond the elementary ones. They need a knowledge of computing to deal with the new age of computers and information systems. They need a knowledge of algebra, geometry, and functions to major in a wide range of fields, from archaeology to zoology. They need a knowledge of statistics for such fields as business, psychology, and economics.
>
> More extensive knowledge and skills, including preparation for calculus, will be needed by college entrants who expect to take advanced mathematics courses or to major in such fields as engineering, economics, premedicine, computer science, or the natural sciences. (College Board, 1983, p. 19)

The reports of this most recent era have been remarkably consistent. They have agreed that many of the problems in mathematics education have to do with the content that is contained in the textbooks and taught in the classroom. There is a strong call for increased attention to real world applications. Since 1983, the reports on mathematics education have continued in this vein. One of the most recent reports was *The Underachieving Curriculum*, which was a report by the International Association of the Evaluation of Education Achievement (IEA) based on results from the Second International Mathematics Study, a study which compared mathematics education in some twenty countries. It called for a restructuring of the mathematics curriculum, a renewed focus on the role of textbooks, and a reexamination and revitalization of the content of the curriculum:

> The domination of the lower secondary school curriculum by the arithmetic of the elementary school has resulted in a program that, from an international point of view, is very lean. The curriculum should be broadened and enriched by including a substantial treatment of topics such as geometry, probability, statistics and algebra, as well as promoting higher-level process goals such as estimation and problem-solving. (IEA, p. 113)

The secondary component of UCSMP had the task of actually creating the grades 7-12 textbooks in line with these recommendations. The director of the secondary component was Zalman Usiskin, a professor of education at the University of Chicago. Currently, he is director of UCSMP and co-director of the secondary component. Usiskin was the author of the first textbook created by UCSMP; the seventh grade text *Transition Mathematics*. He serves as an author or editor of the other five books developed by the secondary component. Until 1988, there was an advisory board solely for the secondary component, and many authors and editors have worked on the UCSMP materials. Usiskin's philosophy and presence, however, dominates the materials.

In trying to achieve the goal of creating textbooks which would demonstrate the applicability of mathematics in the real world, Usiskin was an excellent choice to lead this effort. He was the director of the First-Year Algebra Via Applications Development Project funded by the National Science Foundation (NSF), and in 1976 published the textbook *Algebra through Applications with Probability and Statistics*. The approach of this textbook seemed reminiscent of

the approach towards mathematics of Archimedes. Some lessons were designed to begin with an application and then determine the algebra necessary to model it. Some lessons were simply applications with no new algebra taught. There were also lessons that took a more traditional approach of discussing an algebraic technique first and then applying it. This text was influential in that it showed both the richness of applications that could be brought into the subject of algebra, and the workable process of inducing algebra from an application. The text was never a commercial success although it had its followers and was used in several classrooms.

In *Applications in School Mathematics,* NCTM's 1979 Yearbook, Usiskin's article "Applications in Elementary Algebra and Geometry" explored applications in these courses and where to find them. He summarized his findings:

> Real-world applications of elementary algebra and geometry exist in great numbers. We hope that the ones included here give a flavor of the types of such applications in such a way that the appetite is whetted for more.
>
> With a typical contemporary curriculum, it is easier to do applications in algebra than in geometry. The algebra curriculum does not have to be changed to incorporate the algebra that is most often used in applications. However, the geometry found in applications uses proportionally more coordinates, transformations and three-dimensional ideas than the geometry that usually appears in current geometry courses. (Usiskin, 1979, p. 30)

Later he served on a joint committee of the MAA and NCTM which in 1980 produced the volume *A Sourcebook of Applications of School Mathematics* which included applications from advanced arithmetic through trigonometry and probability. It also included some more lengthy projects for students to do. Immediately preceding UCSMP, Usiskin worked with Max Bell on the Arithmetic and its Applications Project funded by NSF. In 1983, they published *Applying Arithmetic: A Handbook of Applications of Arithmetic.* This three volume work was an organization and categorization of arithmetic applications based on the mathematics inherent in them.

UCSMP chose to develop six different courses beginning in grade 7. The courses following Transition Mathematics are Algebra, Geometry, Advanced Algebra, Functions, Statistics, and Trigonometry with Computers, and Precalculus and Discrete Mathematics. In order to follow the guidelines of the reports since 1975, a number of broad features of the materials run

throughout the entire curriculum. The courses have a wider scope. Reading and problem solving are emphasized throughout. Calculators are assumed, computers are necessary for the fifth course, and computer exercises are in several other courses.

Two other features have important ramifications within the context of addressing the role of mathematics and the real world. The first is that each mathematical idea is studied in detail for its applications to real world problems. The second is that four dimensions of understanding are emphasized: Skills, Properties, Uses, and Representations. UCSMP calls this the SPUR approach.

The SPUR approach is UCSMP's way of addressing the multifaceted aspects of mathematics and mathematics education mentioned within this paper. These four dimensions of understanding are termed dimensions because there is the belief that they are quite independent form one another. One can be proficient in one dimension and not in another. Within a given dimension there can be easy problems as well as difficult ones. A brief explanation of each dimension fully outlines the UCSMP philosophy.

Skills are the mechanics of mathematics. In algebra, skills including equation solving, in geometry, they include drawing and measuring. Many skills are best done on a calculator or computer. The back to basics movement only stressed skills. UCSMP believes that skills are just part of a bigger picture, and while students should master some skills, they should not be limited to them.

Properties are the mathematical underpinnings of concepts or algorithms. In algebra this includes the associative and commutative properties; in geometry, this includes proof writing. It is this aspect of mathematics that the "pure" mathematicians regard as mathematics. It is UCSMP's view that students should receive a taste of abstraction, generalization, and the systematic nature of mathematics.

Uses are, of course, the applications. The two-directional approach is employed, both exploring uses of some given mathematics as well as mathematical modeling of a given situation. Representations are a psychological approach to mathematics. In arithmetic this includes picturing

the algorithms by using sticks or pieces of pie. In algebra and geometry this includes coordinate graphing and categorization schemes such as hierarchies.

In early project materials, a fifth dimension is occasionally mentioned, a historical-cultural dimension. This dimension includes how history shaped the field of mathematics and how mathematics pervades other aspects of the culture. Though this dimension appears in the materials, it is not as prevalent as the others. It is, however, important to the study which follows.

Specifically, UCSMP's approach to incorporating the real world into its materials may be summarized in the following way. It is important that the real part of the real world not be neglected. UCSMP shied away from contrived word problems such as a person rowing a boat upstream for three hours at a constant rate. Who in the real world could perform such a task?

There is also a more subtle contrivance in word problems. For example consider this "standard" coin problem. *If I hold 27 coins which are dimes and quarters and they total $3.75, how many dimes do I have?* It is impossible to know the fact given in the question (the total of $3.75) without knowing the answer to the problem in the first place (20 dimes). These problems, called "reverse given-find", are a continuation of school mathematics meant to "work out" the mind, but they are not an application that someone would really do.

UCSMP chose to put in real situations where the students would have to make decisions using raw, real data. Census data, newspapers, and almanacs are essential sources in UCSMP for creating problems and lessons. There is also the notion that students should know some history of mathematics to realize that the field continues to develop, and that not everything was known to the ancient Greeks.

The proof is in the pudding, and I underwent a study beginning in late 1987 to determine if the rhetoric and goals of UCSMP were actually being carried out in their materials. The study was to compare the UCSMP materials with commercial textbooks to note the way each referred to mathematics and the real world. The study was to determine the number and type of real world contexts in the textbooks.

These real world contexts have now been labeled as "cultural contexts." The real world for a citizen of the United States is different than the real world for a citizen of France or a citizen of the Philippines or a citizen of Lebanon. It is clear that the cultural background of people just within the United States varies tremendously, the real world to a student in the inner city of Chicago is different than that of a Chicago suburban student which is totally different than the real world of a student from rural Minnesota. Therefore, the term "cultural contexts" is a little more accurate than "real world contexts". The culture that the study focuses on in this paper is that of the mainstream U.S. educational system. A person of this culture would have little difficulty understanding the newspaper, television network programming, or commercially published encyclopedias.

A Textbook Comparison Study

The study was a textbook comparison to determine the number and type of cultural contexts in the UCSMP materials compared to other commercial textbooks. *Transition Mathematics* and *UCSMP Algebra* were selected as the UCSMP materials. These two books were selected since they are the first two books in the secondary component's series, and are typical in their approach to mathematics for the UCSMP materials. This also allowed for comparison materials to be both at the middle school and high school levels.

Transition Mathematics was compared against *Heath Pre-Algebra* published in 1986 by D.C. Heath. This text was chosen since its scope was most similar to *Transition Mathematics*. It contains geometry as well as advanced arithmetic. It also provides problem solving and applications. Its target population, moreover, is for seventh or eighth graders who are to take algebra the next year. However, *Transition Mathematics* is aimed at a broader population. A second comparison textbook is *Merrill Mathematics 7*, a standard 7th grade textbook for average students which is part of the publisher's kindergarten through eighth grade series.

UCSMP Algebra was compared against *Algebra 1* published by Houghton Mifflin in 1986. It will be referred to in this paper as Dolciani's *Algebra 1* after its lead author Mary Dolciani. Houghton Mifflin has had the greatest market share of the secondary school textbooks over the past twenty-five years, although the current market is much more competitive. A second algebra comparison textbook was *Algebra 1*, published by Grassdale, which is a controversial and highly publicized text. It is perhaps the best of any back to basics type text. This book has a zealous following, though its approach and author have also undergone harsh criticism. The author, John Saxon, has made clear that his philosophy of mathematics education is to teach skills and to review those skills every day. This text will be referred to as Saxon's *Algebra 1* after the author and the fact that Grassdale Publishing is now Saxon Publishing.

All three algebra textbooks have supplementary material in the textbook. (Material such as extra practice, cumulative reviews, and chapter openers [the first page denoting a new chapter] is not considered supplementary.) Supplementary material is not contained in the lessons and is usually clearly marked as supplementary. While teachers and students most often disregard this material, since it is not considered as a regular and expected part of the algebra course, it was treated as part of the textbook in the study. In *UCSMP Algebra* the supplementary material is the three appendices. In Saxon's *Algebra 1* the supplementary material is four extra lessons at the end of the book. Dolciani's *Algebra 1* contains the most supplementary material. It includes little biographies of mathematicians and mathematics educators (often these were obscure and written in order to include minority representation), tips on reading algebra, college entrance exam question practice, and discussions of some careers or consumer practices.

The other texts differ on supplementary materials. *Transition Mathematics* contains no supplementary materials. Heath's *Pre-Algebra* has materials marked off as applications or careers that are separate from the lessons but included in the chapters. *Merrill Mathematics 7* has these separate materials at the end of a chapter on careers, consumer education, or extensions of the mathematics. It also includes more skill review supplements.

A cultural context was defined as a text paragraph or problem that explicitly referred to a fact in the real world, no matter whether trivial or important. Mathematical facts were disregarded unless they had applicability in common usage outside of mathematics. However, statements that referred to mathematics as a field or the history of mathematics were considered cultural contexts.

The unit of a paragraph or problem was chosen since a paragraph or problem is intended to convey to the reader, or student in this case, a single thought or concept. Any list in the text was considered as one context. A chart, graph, or table was a single context. New vocabulary on its own did not constitute a context unless the word or phrase was actually defined, was not solely mathematical, and the text definition was applicable outside of mathematics. A proper name of a real person, place, or event was considered as an indicator of a cultural context. Obviously fictitious names of people, places, or companies were not considered as contexts, although the use of real names in the same context might have signalled a context. Formulas used outside of mathematics were considered as cultural contexts if what the variables represented was explained. Formulas expressing conversions between standard units of measure were included as cultural contexts. Formulas only used inside mathematics were not considered contexts.

It is best to illustrate why certain statements were contexts and why some were not by using examples. All of these are taken from *Transition Mathematics:*

#26 3,320,000 square miles, the approximate area of the Sahara desert. (p. 82) is considered a context since it gives an actual fact about an actual place.

#29 Leslie used 30% of her $5.00 allowance to buy a magazine. How much did the magazine cost? (p. 82) is not considered a context since it is not known who Leslie is or if this refers to a particular magazine. On contexts using price, it was helpful to have a year given to place the time that the price was valid.

In the United States, science, medicine and photography are almost all metric. (p. 113) is considered a context since it gives a fact about this country. This is one sentence from a full

paragraph. While several sentences in this paragraph could be considered as contexts, the paragraph is counted as one context.

#6 One relationship between the U.S. and metric system is exact. What relationship is it? (p. 114) This is a question, and although the answer might lead to a cultural fact, the question itself does not give out that information. It cannot be considered as a context.

Urban *areas are cities and suburbs. (p. 187)* is defining a piece of vocabulary. Even though this may be known to all students taking the course, it is considered a cultural context.

If an expression contains a variable alone or with number and operation symbols, it is called an ***algebraic expression.*** *(p. 149)* is not considered a context since the phrase defined has its use only in mathematics.

Bar graph of the ups and downs of travel costs. (p. 366) is considered as one context even though seven data points are plotted.

Table of some positive integers and their squares. (p. 263) is not a context since the subject matter is purely arithmetical.

The procedure of the study was a simple one. Each text was read in natural order, and, whenever a cultural context appeared, it was typed into a list. After these initial lists were made, then each context was placed into one of 28 different categories. The list of categories was developed from a list in the article "Applications in Mathematics: An Annotated Bibliography" in *Applications in School Mathematics*. To arrive at the final 28 categories I added some new ones, deleted a few, and changed some of the headings. The lists were then reorganized by categories, and the results are in Appendices A-F (beginning on page 41). Each list proceeds alphabetically by category, and within each category are the contexts in the order in which they appear in the textbook.

The categories included some of the following topics:

Architecture, City Planning: heights of buildings.
Art, Crafts: artists and their history.
Astronomy: includes when the Earth is taken as a planet.
Banking, Money, Taxes: insurance and interest rates.
Biology: zoology, and horticulture.
Business: marketing strategies and the stock market.
Calendar: holidays.
Chemistry: facts about elements.
Consumer Education: nutrition and pricing information.
Demographics: census data and other surveys.
Economics: national statistics such as the Consumer Price Index.
Entertainment: names of celebrities.
Environmental Studies: camping and forestry.
Games: puzzles and some probability.
Geography, Geoscience: distances, maps.
History of Mathematics: mathematicians and when they discovered results or notation.
Industrial Art, Design: catchall category for certain measurements such as dimensions of notebook paper.
Meteorology: weather data, records, and predictions.
Music: mostly history of musicians.
Philosophy of Mathematics: catchall category includes advice and justifications of lesson material.
Physics, Optics: motion, atoms, scientific formulas.
Politics, Law, History: military contexts.
Religion: folklore.
Sports: athletic records, rules, and dimensions of equipment or playing areas.
Surveying, Measurement: conversion formulas.
Technology: calculators and computers.
Transportation: cars, planes, boats, travel, and traffic.
Vocabulary: definitions, foreign words or phrases, and etymology.

A table summarizing the findings is on the next page.

Number of Cultural Contexts in Five Selected Textbooks

	Heath Pre-Algebra (495 pp.)	*Merrill 7 Mathematics* (422 pp.)	*Transition Mathematics* (616 pp.)	Dolciani's *Algebra 1* (698 pp.)	Saxon's *Algebra 1* (407 pp.)	*UCSMP Algebra* (690 pp.)
Architecture, City Planning	3	0	7	3	1	1
Art, Crafts	0	2	10	0	0	0
Astronomy	2	10	14	11	0	20
Banking, Money, Taxes	12	17	15	8	1	19
Biology	14	13	24	4	0	26
Business	5	7	22	4	1	27
Calendar	2	1	4	3	0	5
Chemistry	0	1	3	8	1	6
Consumer Education	19	7	23	3	0	9
Demographics	0	2	46	3	0	54
Economics	3	0	16	0	0	10
Entertainment	5	1	11	0	0	5
Environmental Studies	1	1	2	3	0	3
Games	3	5	3	3	0	20
Geography, Geoscience	7	9	20	9	1	20
History of Mathematics	0	5	31	13	6	10
Industrial Art, Design	0	9	8	6	1	2
Meteorology	16	7	16	10	0	43
Music	1	0	3	0	0	0
Philosophy of Mathematics	0	3	7	6	6	13
Physics, Optics	20	7	13	32	3	26
Politics, Law, History	0	0	15	1	0	22
Religion	0	0	3	0	0	0
Sports	15	6	26	2	0	45
Surveying, Measurement	20	21	61	2	1	23
Technology	2	6	8	6	0	16
Transportation	8	11	17	5	0	20
Vocabulary	1	1	12	8	6	15
TOTAL	159	152	440	153	28	460

The findings in the table lead to some obvious conclusions on first glance and more interesting ones when explored in depth. It is clear that the UCSMP materials have many more cultural contexts in total than the comparison materials, about three times as many. *UCSMP Algebra* has the most contexts, although on a per page basis, *Transition Mathematics* has the most. Just by glancing at the table, it is clear that the UCSMP philosophy of putting in contexts to the real world is carried out in their materials when compared to other textbooks.

While reading the table down the columns is the first order of business, reading the data across the rows is more intriguing. An interesting row is the History of Mathematics. *Transition Mathematics* has 31 of these contexts while one of its comparison texts has none. In reading *Transition Mathematics* with an awareness of these contexts, a nice history of arithmetic and arithmetic notation is included. Then, it seems the history of mathematics is roughly the same in the algebras. However, for Saxon's *Algebra 1*, this is a major category.

The three non-algebra books have many contexts in the category Surveying, Measurement. Again the difference between *Transition Mathematics* and the other texts is easily apparent. It appears that units and unit conversion is considered a pre-algebra topic, as the comparison algebra textbooks have only one context each in this category. Only *UCSMP Algebra* continues to develop this area.

In the comparison textbooks, the scientific categories take up a much larger proportion of the contexts than in the UCSMP materials. These are the more traditional applications. All the textbooks also have a good portion of contexts the the two consumer type categories, which are also traditional.

The UCSMP materials use many more contexts in the non-traditional but equally valid applications of mathematics such as is in demographics; business; economics; politics, law, history; technology; transportation; and vocabulary. Sports is strongly used in *UCSMP Algebra* while ignored in the other algebras, but the non-algebra texts use contexts from this field. This wider range of contexts is indicative that the UCSMP material expose students to more varied uses of mathematics in the culture.

It is interesting to look at three of the least used categories overall: Art, Crafts; Music; and Religion. It is said that mathematics and music are highly related; you could not tell that from these textbooks. Religion is perhaps avoided due to its controversial relationship to the U.S. educational system. The applications to Art are more geometric, and, since *Transition Mathematics* is sold as a pre-geometry as well as a pre-algebra textbook, this is probably why it has more contexts in this category.

Overall, the UCSMP materials, by having more quantity of contexts, are able to have more quality in the diversity of them. *UCSMP Algebra* seems to have the most even distribution of contexts throughout the text, although it could perhaps include more history of mathematics. The UCSMP materials have seemed to do a good job at incorporating into the materials the recommendations guiding the project.

Discussion

In the UCSMP materials it is stressed that students read the textbook, while with comparison materials students most often just read the exercises after listening to a teacher explanation. The contexts in the lessons themselves of the comparison books might be missed in actual usage. In Saxon's *Algebra 1*, exactly 1/4 of the contexts are in footnotes. The other comparison textbooks all have supplementary materials which contain a significant number of contexts. Students seldom get to the last few chapters of the textbook. All of the previous contexts, whether in footnotes, supplementary materials, or at the end of the book, were included as they constitute the course. Over 10% of the contexts in Dolciani's *Algebra 1* appeared in the last chapter, more than any other textbook.

A context was a paragraph or question. It is possible to make a case for a context being a sentence or phrase or even a word. The use of paragraphs as opposed to single words may bias against traditional materials since the vocabulary and context of even contrived problems may have

some meaning. When a person rows upstream at a constant rate, at least the concept of rowing and upstream are being taught. Also, why should a table or chart be counted as one context when each data point may give different information?

It is known in mathematics that when you start from different assumptions different results may follow. On the other hand, it did not make sense to me to include any phrase or word that may be new to the student as a cultural context. It would entail defining what "new" to the student meant. It seems to weaken the concept of a cultural context. There are also many situations such as these included in the UCSMP materials which were, of course, not counted as cultural contexts. The paragraph or question as a unit turns out to have great facility in recording and processing. A word or phrase would make the process more subjective. The overriding criteria for a cultural context was an explicit fact in the real world, not something which might lead to discovering or discussing a real world fact.

There are arguments that address the philosophy of UCSMP. The first of these is that the cultural contexts in the UCSMP materials are simply trivia. They were put in solely to technically please the guidelines of the reports. It is a band-aid solution for a difficult problem. Instead of addressing how mathematics is applied in the real world, a bunch of trivia is included in the materials to distract the users from the fact that this is the same type of mathematics that has been in the curriculum for ages.

While these contexts may be a distraction for some, they are viewed by many as an enhancement to the curriculum. For many students, real situations serve as a motivating force to explore mathematics. For other students real situations are necessary since the only way information learned in school can be processed is to have a real world example to compare it to or fit it into. Psychologists would confirm that some students learn this way as people must fit in new information into already existing mental schema.

A context that might fall into this category is from *UCSMP Algebra: #27 In 1980, the U.S. center of population was 1/4 mile west of De Soto, Missouri. According to American Demographics, the center of population moves 58 feet west and 29 feet south each day. (p. 84)*

Someone who had this information at their fingertips may impress a few people; however, there are interesting things to note solely from this fact. The 1980 center of population is known since censuses are taken every decade in years divisible by ten. De Soto, Missouri, in relationship to the actual center of the continental U.S. would be something to explore. The figures 58 and 29 are very exact. Are they accurate? Will the trend continue? Some interesting fodder for classroom discussion about the culture can emerge from this "trivial" fact as well as some good mathematics on statistical sampling, approximation, growth rates, counting, and arithmetic.

A second global criticism takes the previous argument even further. These cultural contexts are not just trivial, as trivia is relatively harmless; but they are miseducative. They cheapen mathematics from its beauty and its structure. Plato was correct in elevating arithmetica over logistica, for the study of mathematics should be for students to develop the art of generalization, abstraction, and, in today's jargon, higher order thinking skills.

This argument cannot be directly challenged because it represents a philosophic view of what mathematics should be. But the point of view may be addressed with three different comments. One, it is clear that the philosophy of UCSMP states that while abstraction and generalization has its place, it is only one of four major dimensions of understanding. Secondly, one must crawl before one can walk. If these contexts are helpful in sparking students with more interest in mathematics, then their own motivation in later years will lead them into discovering the beauty of its abstractions. Thirdly, the UCSMP materials are geared towards the middle ability students. The people that will develop the field of mathematics from this point on will most likely come from the ranks of the gifted, while it is the middle ability group who should feel comfortable with mathematics in their daily lives: at work, in reading newspapers and literature, in their private domestic lives, and in understanding political, scientific, and academic accomplishments. At present this does not seem to be the case, and it is this problem which UCSMP and its funding organizations seek to correct.

The final critique is not so much a criticism, but a natural query into mathematics education reform. For almost this entire century, a period of some seventy-five years, there has been

pressure and reports stressing that the practicality of mathematics should be addressed in the classroom. Why is it only now that this question is being addressed, or is there, in fact, some overriding force that dooms this approach to failure? Seeing as how there has not been a successful approach to this problem for seventy-five years (as signified by reports calling for the same changes), why should UCSMP expect to succeed where others have only failed?

It is stressed to students that they should learn from their mistakes; this lesson may pertain to the mathematics education community as well. It is clear from the results reported in *The Underachieving Curriculum* that the current curriculum is a failure when compared to other industrialized countries. By viewing results from other countries, it is apparent that some mathematics educational programs are succeeding. The United States, therefore, sees the possibility of a successful school mathematics curriculum. Certainly change must occur for the curriculum to become a success.

It is not clear that all prior curricula were failures. Prior to World War II, many curricula went untested and were aborted without hard data as to their success or failure with students. The "new math" was seen as a failure, but data which may have shown otherwise was not processed. UCSMP has tried to take what it considers to be the best elements of previous curricula to incorporate into its materials. There is a separate and independent evaluation component which tests these materials in the classrooms. *Transition Mathematics* and *UCSMP Algebra* have undergone four and three revisions respectively since their first pilot editions. Many of these revisions are based on results gathered by the evaluation component. This in itself is no guarantee for success, but it shows UCSMP's awareness of the issue. The forces of history are beyond UCSMP's control; so, since it cannot address the aim of teaching the practicality of mathematics at an earlier point in time, it must do so now.

Conclusion: Ideas for Further Study

While a nice case has been made for the inclusion of these cultural contexts in UCSMP materials, it is up to further study to see if these ideas work. Some possible questions for further study are now listed. Does the inclusion of these contexts helps increase students' mathematics achievement? Does incidental learning occur and thus do students have increased achievement in science or social studies? Are students' attitudes towards mathematics affected by the inclusion of these cultural contexts? Are students made more aware of the role of mathematics in the culture or of the history of mathematics?

Further studies might focus on the teachers. Are their attitudes affected by the inclusion of the contexts? Are these contexts discussed in class? Do they provide a springboard for discussion of other subjects such as science, social studies, or current events? How do teachers appraise the UCSMP philosophy towards relating mathematics and the real world? Do they feel that their students benefit from this approach?

Certainly the problem of relating mathematics to the real world has been prevalent throughout the development of mathematics and mathematics education. While UCSMP cannot profess to have solved a problem with major philosophical, historical, and psychological dimensions; it can claim to have taken a position on the issue and to have created materials consistent with that position.

Appendix A: Cultural Contexts in *Heath Pre-Algebra*

Architecture, City Planning

The picture shows the pyramids of the Muttart Conservatory in Edmonton, Alberta, Canada. The volume of these pyramids can be found by using the above formula. (p. 309)

Computers are used to draw "blueprints" of an architect's design. The computer can be programmed to display a floor plan on a screen or printout. (p. 331)

Architects use a blueprint and the formula $E = C \cdot A$ to make an estimate E of the cost of construction based on the cost C per square foot of construction and A sq ft of floor space. (p. 424)

Astronomy

#62 The sun is about 4,500,000,000 years old. (p. 209)

#64 The earth weighs about 11,980,000,000,000,000,000,000,000 pounds. (p. 209)

Banking, Money, Taxes

In many banks you can use a computer-operated teller machine to deposit and withdraw money, make payments, and check the balance of your account . To operate the machine, you insert a special card and enter your personal identification number. Directions will then appear on the screen. (p. 71)

Price chart of Indian Head pennies 1865–1870. (p. 129)

Market value of nickels: 1890, $8.80; 1900, $2.90; 1910, $2.00; 1920, $1.25; 1930, $.75, 1950, $1.75 . (p. 129)

Faces on U.S. currency: Lincoln, $5 bill; Washington, $1 bill; Hamilton, $10 bill. (p. 136)

Sales tax table. (p. 233)

When you put money in a savings account, the bank pays you interest on the principal (the amount in your account). When you borrow money from the bank, you pay the bank interest on the principal (the amount you borrow). (p. 249)

When you borrow money from a bank, credit union, or loan company, you pay for the use of it. The amount you pay is called **interest**. The interest I paid depends on the **principal** p (the amount borrowed), the **rate** r (percent of interest charged), and the **time** t for which the money is borrowed. Simple interest formula: $I = p \cdot r \cdot t$. (p. 270)

When a loan is due, the total amount A owed is equal to the principal p plus the interest prt: $A = p + prt$. (p. 271)

The interest on savings accounts is often compounded annually, semiannually, quarterly, or even daily. In this way, you earn interest on your interest. When the interest is compounded daily, interest is added to the account each day. (p. 273)

$A = P(1 + i)^n$. A = amount in the account; P = original deposit; i = interest rate per year, expressed as a decimal; n = number of years compounded. (p. 273)

Graph of $1000 with interest compounded daily at three different rates over 8 years. (p. 273)

A **budget** is a plan for using one's money. (p. 343)

Biology

You can predict your adult height by first multiplying your present height (in inches) by 100 and then dividing the answer by the growth factor found in the table. (p. 152)
You can use the following formula to find the desired weight w for a given height h of a young adult: $w = \frac{11(h-40)}{2}$. (p. 153)
Chart of recommended daily amounts of food given to dogs. (p. 162)
Top speed for some species of snails is 0.00036 mile per hour. (p. 208)
#60 The thickness of a human hair is about 0.00366 of a foot. (p. 209)
The deepest-diving bird, the emperor penguin, can reach a depth of -870 feet. (p. 215)
The deepest-diving mammal, the bull sperm whale, can reach a depth of -3720 feet. (p. 215)
One hour of dancing burns 2.2 calories per pound. A 120-pound person burns 132 calories during a half hour of dancing, while a 150-pound person burns 165 calories. (p. 223)
Chart of calories burned in certain activities. (p. 245)
Formula to find calories you burn b when exercising: $b = c \cdot h \cdot w$ where c = calories burned per pound in one hour, h = hours of exercising and w = weight in pounds. (p. 245)
#33 Researchers say a man's beard grows an average of 0.038 of a centimeter a day. How many millimeters per day is that? (p. 287)
#34 In one year, a man's beard grows an average of 138 millimeters. How many centimeters per year is that? (p. 287)
#35 During the average lifetime, a man spends approximately 3350 hours removing 838 centimeters of whiskers. How many meters of whiskers is that? (p. 287)
#36 The longest beard belonged to Hans Langseth of Barney, ND. He let his whiskers grow to a length of 5.33 meters. How many centimeters is that? (p. 287)

Business

The amount you pay for a TV is the **retail price.** What the appliance store paid was the **wholesale price.** The difference between the retail price and the wholesale price is the **markup.** (p. 167)
r = w + m; r = retail price, w = wholesale price, m = markup. (p. 167)
There are about 380,000,000,000 telephones in the world. (p. 209)
Chart of stock quotations. (p. 224)
Map and chart of shipping costs from Chicago. (p. 357)

Calendar

#3 Father's Day is always in June. (p. 375)
#4 Valentine's Day is February 14. (p. 375)

Consumer Education

Catalog descriptions of casette player, stereo headphones, casette carrying case, desk top battery charger, and tapes. (p. 11)
Ads for used cars 1976 - 1983 models. (p. 23)
Here is a schedule for direct-dial telephone rates for calls made from Chicago, IL to some other cities (Atlanta, Detroit, Los Angeles, Minneapolis, St. Louis) in the U.S. (p. 39)
Motorcycle for sale Ad. (p. 40)
Apartment ads. (p. 53)
The formula s = r – d relates the sale price s to the regular price r and the discount d. (p. 69)

Ad for film development. (p. 87)
Most families rent campers when they go on a vacation. Most rental firms charge a daily rate plus a certain amount for each mile driven. (p. 91)
picture of yearbook ads. (p. 146)
#1-5 Table of 1975 and 1985 prices of a soft drink, mother's day card, spiral notebook and 45-RPM record. (p. 155)
#6 1985 cost of a movie ticket: $3.50, 1975 cost of a movie ticket: $1.50 . (p. 155)
#7 1985 cost of a burger: $1.20, 1975 cost of a burger: $.50 . (p. 155)
$s = p - d$; s = sale price, p = regular price, d = discount. (p. 167)
Recipe for pizza dough. (p. 174)
Ad "Save trading stamps/Redeem stamps for free gifts". (p. 181)
Plant sale ads. (p. 182)
Chart of TV commercial prices. (p. 219)
Chart of first class letter rate from 1895 to 1985. (p. 375)
A federal excise tax of 2% is added to all station-to-station telephone calls. (p. 437)

Economics

You increase your earnings by increasing your hourly wage, the number of hours worked, or both. A worker's hourly wage is based on several factors, including skill, experience, and the amount of danger involved. (p. 57)
Chart of Regular Gasoline prices in the U.S. 1973–1980. (p. 61)
Your earnings e depend upon the number of hours h you work and your hourly wage w: $h \cdot w = e$. (p. 83)

Entertainment

Alan Alda, Ringo Starr, Harry Belafonte, Paul Newman, Elvis Costello. (p. 2)
Pictograph of TV Survey: Academy Awards, Super Bowl, Emmy Awards, Rose Bowl Parade. (p. 32)
Lionel Richie concert. (p. 50)
If you divide Michael Jackson's age when he first sang in public by 7 and then add 19, you get 20. (p. 104)
Walt Disney–cartoonist, Chris Evert-Lloyd–tennis player, Eisenhower–U.S. president, Donna Summer–singer. (p. 130)

Environmental Studies

Trail rules and information chart. (p. 178)

Games

When you roll two dice, there are 36 possible outcomes. Therefore, the probability of rolling a 4 and a 6 or a 6 and a 4 is $\frac{2}{36}$, or $\frac{1}{18}$. (p. 361)
#45 When you toss a thumbtack, there are these two possible outcomes: point up, point down. (p. 367)
#46 When you toss a paper cup, there are these three possible outcomes: bottom down, top down, side down. (p. 367)

Geography, Geoscience

Mileage chart of cities: Baltimore, Chicago, Cleveland, Denver, Houston, Los Angeles, New York, St. Louis. (p. 191)

The water level of the ocean rises about 0.0029 of a foot every year. (p. 209)

A map is an example of a scale drawing. (p. 261)

Map of slice of U.S. from Des Moines to Boston and south to Pittsburgh. (p. 261)

The actual air distance from Chicago to Boston is 1352 kilometers. (p. 261)

The photograph above was taken in Nebraska, south of Morrill. This giant irrigation circle has a radius of about 500 yards. You can use this fact and the formula above to estimate the area of the circle. (p. 277)

Map of Florida with distances from Orlando (home of Disney World): Tallahassee, 355 km; Jacksonville, 196 km; Tampa, 125 km; Miami, 336 km. (p. 285)

Meteorology

On February 6-7, 1978, a record-breaking snowfall occurred in Boston. A total of 30 inches of snow fell during a 24-hour period. (p. 145)

The formula $w = \frac{s}{8}$ relates inches of water w to inches of snow s. (p. 149)

Chart of Extreme temperatures of selected U.S. cities in 1980. (p. 196)

#47 The relationship between a temperature in degrees Celsius C and the temperature in degrees Fahrenheit F is given by the following formula: **9C = 5f – 160**. Substitute in the formula above to complete this chart. (p. 197)

#58 An average-size thundercloud holds about 6,000,000,000,000 raindrops. (p. 209)

#61 Lightning strikes the earth about 8,640,000 times every day. (p. 209)

The highest temperature in Chicago was 104°F. The lowest temperature in Chicago was -19°F. (p. 216)

#3 A record for extreme temperatures in a 24-hour period was set in Browning, MT, in 1916. The temperature dropped 100 degrees from 44°F to -56°F. (p. 217)

#4 A most unusual rise in temperature occurred on January 22, 1943, in Spearfish, SD. From 7:30 A.M. to 7:32 A.M. the temperature rose 49°F from -4°F to 45°F. (p. 217)

You can estimate the temperature by counting a cricket's chirps. If a cricket chirps 60 times per minute, the temperature is about 55°F. (p. 335)

Nature provides us with many interesting number relationships. For example, the number of times a cricket chirps per minute x depends on the temperature in °F y: $y = \frac{1}{4}x + 40$. (p. 347)

The photo shows the Juneau Icefield in the Coast Mountains of Alaska. The temperature at the time the photo was taken was -20°C. You can use the computer program on page 431 to convert this temperature to degrees Fahrenheit. (p. 421)

Wind-Chill Chart. (p. 431)

#7 The lowest temperature recorded in Nova Scotia, Canada is -41°C. How many degrees Fahrenheit is that? (p. 431)

#8 The lowest temperature recorded in Yukon Territory is -63°C. How many degrees Fahrenheit is that? (p. 431)

#9 The highest temperature recorded in Alberta, Canada is 42°C. How many degrees Fahrenheit is that? (p. 431)

Music

Wolfgang Amadeus Mozart is a famous eighteenth-century composer. (p. 105)

Physics, Optics

Airplane pilots use information about jet streams to arrive at their destination in the shortest amount of time with the most efficient use of fuel. (p. 1)

$a + t = g$: Airspeed a plus tailwind speed t equals ground speed g. (p. 19)

The gas mileage of a vehicle depends upon many factors, the most important of which is size. The average gas mileage for an automobile is about 15 miles per gallon, while the average gas mileage for a cargo vehicle is about 9 miles per gallon. (p. 27)

Gas mileage is the numbers of miles driven for each gallon of gasoline. Formula $\frac{d}{g} = m$ where d = distance, g = gallons, and m = mileage. (p. 49)

The distance d that you travel is equal to rate r multiplied by the time t: $d = rt$. (p. 99)

In an emergency, a driver might have to stop quickly. The braking distance of a car is the distance traveled after the brakes are applied and until the car comes to a stop. The braking distance depends upon the type of road surface and the weather conditions. (p. 101)

Graph of braking distances for automobiles on wet and dry concrete roads at different speeds. (p. 101)

#5 The braking distance on wet asphalt is 1.32 times the braking distance on dry asphalt. (p. 101)

#6 The braking distance on dry concrete is half the braking distance on packed snow. (p. 101)

The distance d in feet that an object will fall in t seconds is given by the following formula: $d = 16t^2$. (p. 121)

The height h in miles at which a pilot must fly in order to see a distance of d miles is given by the following formula: $h = \frac{d^2}{8000}$. (p. 125)

Chart relating height of a balloon to distance seen from balloon. (p. 160)

A regular pencil can draw a line 2,210,000 inches long before it runs out. (p. 208)

#63 It takes 0.00025 of a second for a stick of dynamite to detonate. (p. 209)

The reaction distance is the distance that you travel during the time it takes you to move your foot from the accelerator to the brake pedal. The formula $d = \frac{11s}{10}$ shows the relationship between reaction distance d in ft and speed in mph. (p. 231)

When you push down or let up on the accelerator of a car, the car changes its speed. The change in speed that the car experiences during a period of time is called **acceleration**. To find acceleration, you can use this formula: $a = \frac{f - s}{t}$ where a = acceleration, f = final speed, s = starting speed and t = period of time. (p. 242)

When acceleration is negative, it is called **deceleration**. (p. 242)

The approximate distance d in feet that it takes you to stop a car traveling at the rate of r miles per hour is given by the formula: $d = \frac{6r^2}{100}$. (p. 265)

The higher you go, the farther you see. The chart shows how far you can see on a clear day at different heights. (p. 417)

$d = 89.4\sqrt{h}$ where d = distance seen in miles and h = height in miles. (p. 417)

Sports

Bird, Parish, Maxwell, Kite, Johnson, McHale, Buckner, Ainge. (p. 4)

1984 Olympics Gold Medal winners: Joan Benoit, Mary Lou Retton, Greg Louganis, Carl Lewis. (p. 28)

100-yard dash. (p. 58)

In drag racing, the car that travels a quarter of a mile in the shortest time wins the race. (p. 62)
The *elapsed time* is the time in seconds that the car took to travel a quarter of a mile. The *terminal speed* is the speed (in miles per hour) of the car when it reached the finish line (p. 62)
Jim Knaub broke the Boston marathon wheelchair record on April 18, 1983. He completed the 26.2188 mile course in 1.8 hours. Knaub, a 27-year-old resident of Long Beach, CA, has been paralyzed since 1977. The former Cal State Long Beach pole-vaulter and later its track coach attributes his win to a more efficient wheelchair and improved race conditions. (p. 78)
States with great skiing: California, Utah, Vermont, Colorado. (p. 133)
Basketball-Scoring Formula: $T = 2g + f$ where T = total points, g = field goals, and f = free throws. (p. 141)
Walter Payton, a leading rusher in the NFL, broke the record formerly held by Jim Brown for total lifetime yards gained. However, Jim Brown still holds the record for greatest average rushing yardage, 5.2 yards. (p. 195)
The formula $a = \frac{t}{r}$ where a = yards per carry, t = total yards gained or lost, and r = number of times the ball was carried indicates a football player's average rushing average. (p. 213)
Picture of 24 NFL helmets. (p. 258)
Dimensions of tennis court: 11 m by 23.8 m; perimeter = 69.6 meters. (p. 288)
#9 The bases on a baseball diamond are 90 feet apart. How long a throw is it from second base to home plate? (p. 415)
You could use the formula $A = \frac{H}{(B - W)}$ to compute a player's batting average A based on H number of hits, B number of times at bat, and W numbers of walks. (p. 426)
The program at the right computes and prints a bowler's handicap, which is 80% of the difference between a bowler's average and 200. (p. 433)

Surveying, Measurement

caption with 24 hours = 1 day and 365 days = 1 year. (p. 36)
$1000 bag of dimes weighs between 48 and 49 pounds. (p. 42)
Chart with 1 yard (yd) = 3 feet (ft), 1 ft = 12 inches (in.), and 1 yd = 36 in. (p. 184)
Chart with 1 day = 24 hours (h), 1 h = 60 minutes (min), and 1 min = 60 seconds (s). (p. 185)
Chart with 1 gallon (gal) = 4 quarts (qt), 1 qt = 2 pints (pt), and 1 pt = 2 cups(c). (p. 185)
A **protractor** is used to measure an angle. (p. 278)
Chart of units used to measure length in the metric system. (p. 284)
In the metric system, the **meter** (m) is the basic unit of length. (p. 284)
From the top of the Ferris wheel, a person can see about 5 city blocks–that's about 1 **kilometer**. (p. 284)
The length of a seat on the Ferris wheel is about 1 **meter**. (p. 284)
The width of an index fingernail is about 1 **centimeter**. (p. 284)
The thickness of an eyeglass lens is about 1 **millimeter**. (p. 284)
Chart with 10 mm = 1 cm; 100 cm = 1 m; 1000 mm = 1 m; and 1000 m = 1 km. (p. 286)
The **liter** (L) and **milliliter** (mL) are metric units of liquid measure or capacity: 1 L = 1000 mL. (p. 324)
The **gram** (g) and **kilogram** (kg) are metric units of weight: 1 kg = 1000 g. (p. 324)
#38 A marshmallow weighs about 5 grams. How many marshmallows are there in a 0.45-kilogram bag of marshmallows? (p. 325)
#40 A teaspoon has a capacity of 5 milliliters. How many teaspoons of vanilla are there in a liter bottle? (p. 325)
A container that holds 1 cubic centimeter of water has capacity of 1 milliliter and the weight of the water is 1 gram. (p. 326)

A container that holds 1000 cubic centimeter of water has capacity of 1 liter and the weight of the water is 1 kilogram. (p. 326)
#5 Hint: 5280 feet = 1 mile. (p. 417)

Technology

To communicate with a computer, you have to use a language that the computer understands. A commonly used language is Beginner's All-purpose Symbolic Instruction Code, or **BASIC**. (p. 422)
Before writing a program, a programmer may write a plan called a **flowchart**. (p. 438)

Transportation

Antique car auction: 1933 Mercedes, 1936 Packard, 1908 Stanley Steamer, 1936 Ford convertible. (p. 6)
The greatest collection of antique cars is the Harrah Collection of 1700 cars, estimated to be worth $4 million. (p. 7)
From a height of 528 feet ($\frac{1}{10}$ mile), a pilot can see a distance of about 28 miles. This means that a helicopter pilot can give traffic reports for an entire metropolitan area. (p. 119)
Cross-country bicyclists use the formula $r = \frac{d}{t}$ to calculate how fast they must travel to reach a certain destination in a given amount of time. (p. 171)
The *Douglas Chicago* (Wingspan: 50 ft, Length: 35.5 ft) was flown in 1924 by a team of U.S. Army pilots who were the first to fly around the world. (p. 252)
The *Lockheed Vega* (Wingspan: 41 ft, Length 27.5 ft) was flown in 1932 by Amelia Earhart who became the first woman to fly across the Atlantic solo and nonstop. (p. 252)
Two questions from a driver's license test. (p. 264)
The slope, or grade, of a road is the ratio of the "rise" to the "run." (p. 401)

Vocabulary

Percent means "per 100." (p. 256)

Appendix B: Cultural Contexts in Merrill's *Mathematics 7*

Art, Crafts

She makes different kinds of candles by using different kinds of wax. (p. 152)
She used a scale to measure the amount of clay. (p. 156)

Astronomy

#23 There are more than 2×10^{20} stars in the universe. (p. 100)
#24 Earth is 9.296×19^7 miles from the sun. (p. 100)
#39 The star Proxima Centauri is about 40,000,000,000 kilometers from Earth. (p. 101)
#40 Uranus is about 2 billion miles from the sun. (p. 101)
#42 Saturn is about 900,000,000 miles from the sun. (p. 101)
#43 Mars is about 141,700,000 miles from the sun. (p. 101)
#5 Earth is 150 Gm from the sun. Find the distance in meters. (p. 237)
#6 Neptune is 2.8 Tm from the sun. Find the distance in meters. (p. 237)
During a countdown, *negative* numbers are used to represent how much time remains before launch time. (p. 362)
When Apollo 11 started its orbit of the moon, it had traveled 394,337 kilometers in 3 days, 3 hours, and 50 minutes. (p. 371)

Banking, Money, Taxes

This amount is paid by check (with picture of check). (p. 4)
She uses a deposit slip like the one shown. (p. 76)
A passbook, shown below, is used to record deposits, withdrawals, interest, and balance in a savings account. (p. 77)
When giving change, always had the customer the coins first, then the bills. (p. 78)
Money amounts are usually rounded in a certain way. (p. 89)
A **finance charge** is the amount the store charges you for the right to make installments. (p. 110)
#39 A quarter is $\frac{1}{4}$ of a dollar. How many quarters are in six dollars? (p. 207)
#40 A dime is $\frac{1}{10}$ of a dollar. How many dimes are in three dollars? (p. 207)
A sample check is shown below. (p. 302)
Before using checks, a person must **deposit** money into a **checking account**. (p. 302)
Most people use a check register, like the one shown below, to record their checks and deposits. (p. 302)
When you deposit money in a savings account, the bank pays you **interest** on the money. (p. 326)
interest = principal x rate x time. (p. 326)
If you get a loan from a bank, you must pay interest on the amount of the loan. (p. 327)
When a bank adds interest to a savings account, the interest earned becomes part of the principal. (p. 333)
This rate, called the **annual yield**, is greater than the interest rate. (p. 333)
Now most banks use computers to compound interest by the day or by the minute. (p. 333)

Biology

#65 The average person's heart beats 259,200 times in 60 hours. How many times does the heart beat each hour? (p. 49)

The largest squash ever grown measured about 318 centimeters around. (p. 98)

#25 The pterodactyl existed 1.8×10^8 years ago. (p. 100)

#26 The fossil cephalopod is thought to be 4×10^8 years old. (p. 100)

#41 The early horse is thought to have lived 40,000,000 years ago. (p. 101)

#44 The trilobite fossil is thought to be 600,000,000 years old. (p. 101)

#11 To the nearest whole number find the normal blood pressure (p) for a person your age (A). Note that $P = 110 + 0.5A$. (p. 164)

A worker honeybee collects enough nectar in tis lifetime to make about $\frac{1}{10}$ pound of honey. (p. 208)

An American black bear weighs about $\frac{3}{8}$ as much as a grizzly bear. (p. 214)

#1 An American black bear's average life span is 20 years. It takes a cub $\frac{1}{10}$ of his life to become an adult bear. How many years does it take a cub to become an adult? (p. 214)

#25 A breath of air contains about 0.6 L. The average person takes 17 breaths each minute. How many liters of air does a person take in each minute? (p. 239)

#26 For every heartbeat, about 6 mL of blood pass through the heart. Suppose your heart beats 70 times each minute. How many milliliters of blood pass through your heart each minute? (p. 239)

Weight loss chart. (p. 245)

Business

#54 The commercial forests in the U.S. contain about 605,000,000,000 cubic feet of wood. Write this number in expanded form three ways. (p. 31)

#7 The trans-Alaska pipeline can deliver about two million barrels of oil a day. The 810-mile trip takes about five days. How many miles a day is this? (p. 32)

Coal makes up $\frac{4}{5}$ or $\frac{8}{10}$ of the energy reserves in the United States. (p. 168)

A tree 18 inches in diameter and 80 feet tall will produce $\frac{7}{10}$ cord of firewood. (p. 182)

In the United States, about one-fourth of all motor vehicles sold are trucks or buses. (p. 298)

Many salespeople earn **commission** on the total amount of their sales. (p. 330)

Newspapers include much information that is related to probability and statistics. (p. 334)

Calendar

#3 choosing a date in May (with picture below). (p. 335)

Chemistry

Some properties of water are as follows: expands when frozen; can be a solid, liquid, or gas. (p. 32)

Consumer Education

The water meter measures the amount of water used in Teresa's home. The water is measured in cubic feet. (p. 24)
An electric meter measures the amount of electricity used in kilowatt hours. (p. 24)
The order form and tables are shown. (p. 54)
Each pound of round steak has 0.08 pounds of fat. (p. 88)
Many supermarkets provide **unit prices**. (p. 93)
Contents of 1 glass whole milk. (p. 248)
A **discount** is an an amount *subtracted* from the regular price. (p. 316)

Demographics

In a recent year, there were about 1,500,000 farmers in the United States. (p. 100)
Predictions about large groups of people are often made by studying smaller groups, or **samples**. (p. 349)

Entertainment

A record that sells one million or more copies is awarded a golden record. The **standard form** for the whole number one million is 1,000,000. (p. 2)

Environmental Studies

Each day there are 8 tours at Hidden Caverns. (p. 46)

Games

When a penny is tossed, there are two possible outcomes, heads (H) or tails (T). (p. 335)
The possible outcomes for a roll of two dice are listed below. (p. 338)
Each coin can come up either heads (H) or tails (T). (p. 341)
When a coin is tossed, the probability of *heads* is known to be $\frac{1}{2}$. (p. 347)
#1 Suppose you roll a die 150 times. How many times would you expect to roll a 5?
Hint: $P(5) = \frac{1}{6}$. (347)

Geography, Geoscience

The area of Sitka National Historical Site in Alaska is about 100 acres. (p. 30)
The Americas use $\frac{2}{5}$ of the energy used in the world. Russia uses $\frac{3}{20}$ of the energy used in the world. (p. 188)
Table of world-wide energy use. (p. 189)
On the map of California, 1 cm represents 90 km. (p. 292)
The actual distance from Los Angels to Eureka is about 945 km. (p. 292)
Most globes and maps show grid lines, called latitude lines and longitude lines. (p. 386)
Latitude and longitude are measured in degrees because circles may be divided into degrees. (p. 386)

Any point on Earth is a certain number of degrees north or south of the equator. (p. 386)
Any point on earth is within 180° east or west of the prime meridian. (p. 386)

History of Mathematics

We use the Hindu-Arabic numeration system. It was first used by the Hindus in India about 200 B.C. The zero was invented around A.D. 600. About A.D. 800, the Arabians adopted the system. However, this system was not standardized in Europe until around 1450. (p. 5)
In ancient Egypt, the symbols at the right had the values shown. The numeral 1,234 is expressed as follows. (p. 9)
The Roman numeration system is very old but is still in use. Some Roman numerals and their values are shown. (p. 21)
Lattice multiplication is so named because it looks like latticework on windows. This method was used until about 1500.
William Shanks spent 15 years calculating π to 707 decimal places. (p. 257)

Industrial Art, Design

Each sheet of paper in a textbook is about 0.008 centimeters thick. (p. 89)
Many of the shapes studied in geometry are used in the design of hang gliders. (p. 114)
Airport runways suggest perpendicular and parallel lines. (p. 120)
The windows of an airplane have the same size and shape. (p. 122)
Simple, closed figures were used in the design of this vehicle that landed on the moon. (p. 124)
Most helicopters are built using triangular shapes. (p. 128)
One of the shapes often found on a satellite is that of a quadrilateral. (p. 130)
A piece of metal is turned on a lathe. (p. 194)
The top and bottom of the lampshade at the left are shaped like circles. (p. 256)

Meteorology

Two thermometers are shown below. The one on the left measures temperature in degrees Celsius (°C). The one on the right measures temperature in degrees Fahrenheit (°F). (p. 250)

To change Celsius to Fahrenheit, use the formula $F = \frac{5}{9}C + 32$. (p. 250)

To Change Fahrenheit to Celsius, use the formula $C = \frac{5}{9}(F - 32)$. (p. 250)

"There is a 30% chance of rain today." This means that the probability of rain is 30% or $\frac{3}{10}$. If the same report is given on 10 days, it is likely to rain on three of the days. (p. 339)
Any National Weather Service office can see an up-to-date weather map of the nation at any time. (p. 383)
The weather information is also used to make forecast maps. (p. 383)
Scientists do not know all the possible ways that weather is affected. (p. 383)

Philosophy of Mathematics

Pictographs and **bar graphs** make it easy to compare quantities. (p. 350)
Line graphs are used to show growth or change. (p. 352)
Circle graphs allow you to easily compare parts of a whole or total. (p. 352)

Physics, Optics

For example, the formula $d = 16t^2$ shows the distance (d) in feet a free-falling object falls in t seconds. (p. 164)
The formula for voltage is V = IR. In the formula, V represents voltage, I is the current in amperes, and R is the resistance in ohms. (p. 164)
The **density** of a substance is the mass per unit of volume. To find the density, divide the mass in grams by the volume in cubic centimeters. (p. 278)
When an object is placed in water some of the water is *displaced*, or moved. (p. 278)
#2 The mass of a copper penny is 3.1 g and the volume is 0.35 cm^3. Find the density. (p. 278)
#3 The density of water is 1.0 . find the mass of 32 mL of water. (p. 278)
#4 The density of pure gold is 19.3 . find the mass of 10 cm^3 of gold. (p. 278)

Sports

#64 Baseballs are packed 24 to a box. How many boxes are filled by 1,000 baseballs? (p. 49)
For each Olympic event, three medals are awarded; gold silver and bronze. In one event, the United States won two of three or two-thirds of the medals. (p. 58)
Eric Heiden won five gold medals at the 1980 Winter Olympics. His time in the 5,000-meter speed skating event was 7 minutes $2\frac{29}{100}$ seconds. (p. 60)
As a member of the Olympic team, Mary Meager swam the 200-meter butterfly. (p. 62)
#65 Chart B lists winning times in the women's Olympic 200-meter dash. (p. 63)
Edwin Moses set an Olympic record for the 400-meter hurdles. His official record time is 47.64 seconds. (p. 64)

Surveying, Measurement

#10 Mrs. Harris used 496 kilowatt hours (kWh) of electricity in May, 524 in June, and 610 in July. How many more did she use in July than in May? (p. 53)
In one hour, a black and white television uses $\frac{1}{4}$ kilowatt-hour (kWh) of electricity. (p. 174)
#79 An electric bill for 31 days is $34.40. Find the cost per kilowatt hour (kWh) to the nearest cent if 370 kWh are used. (p. 179)
She found examples of length and distance units related to her bike (chart follows). (p. 230)
Measurements are not exact. They are approximate. (p. 234)
#31 A sheet of notebook paper is 0.08 mm thick. Find the thickness of 45 sheets of notebook paper to the nearest millimeter. (p. 235)
#32 A sheet of construction paper is 0.2 mm thick. Find the thickness of 12 sheets of construction paper to the nearest millimeter. (p. 235)
1 gram is about the mass of a large paper clip. (p. 236)
1 kilogram is about the mass of this book. (p. 236)
#26 The total mass of 50 pennies is 155 g. Find the mass of one penny in grams. (p. 237)

#28 A dime has a mass of 2.27 grams. A quarter has a mass of 5.67 grams. How much greater is the mass of the quarter? (p. 237)
#31 A nickel has a mass of 5 grams. Helen has a roll of nickels worth $2.00. Find the total mass of the nickels in grams. (p. 237)
Table of more lesser known metric prefixes. (p. 237)
1 milliliter is about the capacity of an eyedropper. (p. 238)
Weight conversion table. (p. 244)
Liquid measurement table. (p. 245)
Time conversion table. (p. 246)
#40 A stack of 12 pennies is 18 mm thick. How thick is one penny? (p. 252)
The rate 8 km/h is read *8 kilometers per hour*. (p. 284)
The height of a tree or building can be measured indirectly by using shadows. (p. 291)
You can use similar triangles to estimate the height of a flagpole. (p. 295)

Technology

A computer is a system which can store, update, manipulate, and use information. (p. 136)
Box explaining parts of a computer. (p. 213)
The stopwatch shown at the left will measure lengths of time up to one hour. (p. 253)
Computers are used by the National Weather Service. (p. 383)
A computer can do many tasks very quickly, but it cannot think. (p. 392)
Instructions to a computer must be written in a computer language. (p. 392)

Transportation

Mileage Chart including Chicago, Dallas, Denver, Houston, and Los Angeles. (p. 13)
Mileage Chart including Atlanta, Detroit, Memphis and Seattle. (p. 19)
Mileage Chart including Chicago, Dallas, Los Angeles, and New York. (p. 25)
Mileage Chart including Amarillo, El Paso, and Houston. (p. 26)
Time and distance map of part of the U.S. (p. 52)
The most direct route from New York City to St. Louis is through Columbus. (p. 52)
It is about 964.6 miles driving from Portland, Oregon to Los Angeles. (p. 68)
Mileage Chart including Harrisburg, Philadelphia, Pittsburgh, and Washington, D.C. (p. 83)
About one-fourth of the energy used in the United States is for transportation. (p. 184)
Airplanes, buses, and trains use $\frac{1}{4}$ of the transportation energy. (p. 184)
About $\frac{7}{8}$ of the automobiles made in the United States have power brakes. (p. 300)

Vocabulary

Many of the terms used in mathematics come from Latin words. (p. 171)

Appendix C: Cultural Contexts in *Transition Mathematics*

Architecture, City Planning

#22 A famous building just out side Washington, D.C., is pictured below. A clue to its name is given by its shape. What is its name? (p. 228)
#37 The photograph below pictures balusters (the parallel vertical supports) and a banister (the transversal) on stairs. Give two other places where it is common to find parallel lines and transversals. (p. 317)
Picture of Independence Hall, Philadelphia. (p. 337)
#26 Tiles on floors or ceilings usually form tessellations. Find at least two examples. Sketch the shape of the fundamental region in each. (p. 375)
#27 If you visit the Alhambra you will see many examples of tessellations. (p. 375)
#7 The tallest building in the world is the Sears Tower in Chicago. The gift shop at the top of the tower sells models of the tower that are 0.0002 times actual size. The height of the model is 11 cm. (p. 448)
#9 Below is a photograph of two of the great pyramids of Giza in Egypt. Each face has a base about 230 meters long. The height of each face is about 92 meters. What is the area of a face? (p. 587)

Art, Crafts

Picture of Jan van Eyck's *Portrait of a Man in a Red Turban*. (p. 137)
#45 The quilt pattern below is made up of 16 small squares. How many bigger squares are in this pattern? (p. 210)
Picture of The Art Institute in Chicago. (p. 341)
#21 An artist who used translations is M.C. Escher. Many libraries have books of his work. Find such a book and, from it, trace an example of a figure and its translation image. (p. 361)
Picture of *Self-portrait* by M.C. Escher. (p. 361)
The Dutch artist Maurits Escher became famous for the unusual shapes he used in tessellations. (p. 373)
A tessellation by M.C. Escher. (p. 373)
In 10-13, trace the figure onto hard cardboard. Make a tessellation using the figure as a fundamental region. Figure 13 is from the drawings of Maurits Escher. (p. 374)
Example 4 Dollhouse models are often 1/12 actual size. A window that is 18 mm wide on the model is how wide in reality. (p. 447)
#4 some dollhouse furniture is 1/12 actual size. If a dollhouse table is 3.25 inches long, how long is the actual table? (p. 448)

Astronomy

Example 1 The speed of light is nearly 186,281.7 miles per second. (p. 18)
The distance from Earth to a galaxy of stars such as this spiral galaxy is measured in terms of light-years. One light-year is equal to about ten trillion kilometers. (p. 55)
Light travels at a speed of about 186,281.7 miles per second. (p. 62)
The speed of light is 16,094,738,880 miles per day. (p. 63)
Example 1 The distance from the Earth to the Sun is about 150,000,000 km. (p. 63)

#1 $2.6 \cdot 10^{13}$ miles is the approximate distance to the nearest star (other than the Sun), Alpha Centauri. Write this number as a decimal. (p. 64)
#7 5,880,000,000,000,000,000,000 tons, the approximate mass of the Earth. (p. 65)
#31 93,000,000 miles is the approximate distance from the Earth to the Sun. Write this number in scientific notation. (p. 86)
Topographic map of surface of Venus. (p. 384)
#20 The distance from the earth to the moon averages about two hundred thirty-nine thousand miles. Write this number in scientific notation. (p. 608)
Planets and moons in space, baseballs, marbles, and many other objects are in the shape of spheres. (p. 609)
Example 1 The Earth is nearly a sphere with radius of 3960 miles. What is its surface area? (p. 610)
#14 The moon is approximately a sphere with radius 1080 miles. (p. 611)
#32 The Earth goes around the Sun in an orbit that is almost a circle with radius 150,000,000 km. How far does Earth travel in one year in its orbit? (p. 617)

Banking, Money, Taxes

#11 The U.S. Internal Revenue Service allows taxpayers to round all amounts to the nearest dollar. But half dollars must be rounded up. In figuring income tax, to what value can you round each amount? (p. 20)
#44 Money rates are often given as percents. (p. 73)
#21 An interest penalty is charged on credit card purchases if you do not pay on time. (p. 77)
#27 Newspaper columnist Georgie Anne Geyer once wrote about receiving a tax bill for \$0.01 . (p. 89)
#31 The United States was the first country in the world (1786) to have a money system based on decimals. (p. 111)
#31 c On September 17, 1983, UPI reported that a truck loaded with 7.6 million new pennies overturned on Interstate 80 in the mountains north of Sacramento, California. How many dollars is that? (p. 111)
#39 some money equivalencies given. (p. 112)
In 13 and 14, the formula $c = 20n + 5$ gives the cost of first-class postage in 1988; c is the cost in cents; n is the weight in ounces of the mail, rounded up to nearest ounce. (p. 181)
Examples of negative numbers are deposits and withdrawals in a savings account, ups and downs of temperatures or weight, profits and losses in business, and gains and losses in football or other games. (p. 198)
#12 In 1988, it cost 25¢ to mail a one-ounce first-class letter. It cost 20¢ for each additional ounce or part of an ounce up to 11 ounces. (p. 262)
Bar graph of U.S. savings and loan associations earnings 1979–1983. (p. 338)
#29 First-class postage in 1988 cost 25¢ for the first ounce and 20¢ for each additional ounce. (p. 477)
The costs of mailing a first-class letter in 1988 are in the table below. (p. 556)
Postal rates. 1965, $c = 5w$; 1975, $c = 11w + 2$; 1988, $c = 20w + 5$ where w = weight in ounces and c = cost in pennies. (p. 557)
#11 A quarter has radius 12.15 mm. What is the area of one of its faces? (p. 602)

Biology

#22 Consider the following sentences. Each human hand has 27 small bones. Together the hands have over $\frac{1}{4}$ of the 206 bones in the whole body. (p. 41)

#2 $1.6 \cdot 10^5$ kg is the approximate weight of the largest blue whale ever measured. Write this number as a decimal. (p. 64)

#24 An electron microscope can magnify an object 10^5 times. The length of a poliomyelitis virus is $1.2 \cdot 10^{-8}$ meter. Multiply this length by 10^5 to find how many meters long the virus would appear to be when viewed through this microscope. (p. 86)

Picture of poliomyelitis virus under magnification. (p. 86)

#11 $2 \cdot 10^{-7}$ meter, the length of the longest known virus. (p. 89)

#29 It is estimated that a swarm of 250 billion locusts descended on the Red Sea in 1889. Write this number in scientific notation. (p. 92)

#54 The number of non-human living things on Earth is estimated at $3 \cdot 10^{33}$. Write this number as a decimal. (p. 94)

#22 An adult human brain weighs about 1.5 kg. Convert this to ounces. (p. 114)

In 19-21, the picture is a closeup of the markings on a giraffe. a. Tell whether the angle is acute, right, or obtuse. b. Measure the angle. (p. 124)

These chicks are working from inside their shells to free themselves. (p. 158)

#51 The formula $I = 100m/c$ is sometimes used to measure a person's IQ. The IQ is I, mental age is m, and chronological age is c. What is I if $m = 7$ and $c = 5.5$? (p. 185)

#21 A dinosaur known as Brontosaurus weighed about 60,000 pounds. Write this number in scientific notation. (p. 197)

Picture of an Arizona roadrunner. (p. 293)

Graph titled indoor greenery about house plants. (p. 352)

Symmetry occurs often in nature. (p. 369)

Pictured here are honey bees in their beehive. (p. 372)

In 15-18, use the graph below. It shows average heights of boys and girls for the ages 12–16. (p. 380)

As the zebra colt grows, its size expands. (p. 408)

#11 A microscope lens magnifies 150 times. Some human hair is about 0.1 mm thick. How thick would the hair appear under this microscope? (p. 411)

#11 The actual damselfly is half the size of the damselfly pictured at left. How long is the actual damselfly? (p. 415)

#8 A newborn giraffe is 1/3 the height of an adult giraffe. If the newborn is 6 feet tall, how much taller is the adult? (p. 448)

#25 Robert Wadlow of Alton, IL, was $5\frac{1}{3}$ feet tall at the age of 5 and $8\frac{2}{3}$ feet tall at the age of 21. On the average, how fast did he grow in these years? (p. 514)

#22 A plant called the lichen grows 0.01 inch in a year. How many inches will it grow in a century? (p. 603)

#16 Make a circle graph from the information provided here. Of all adults: 22% sleep 6.5 hours or less, 26 % sleep 6.5 to 7.5 hours, 37% sleep 7.5 to 8.5 hours, 9% sleep 8.5 to 9.5 hours, 6 % sleep 9.5 hours or more. (p. 611)

Business

#10 Stock Market: no change, gain, loss. (p. 24)

#40 New car sales graph. (p. 73)

#53 The value of a one-carat flawless diamond reached $64,000 in 1980. By July of 1988, the price had dropped 34% of its former value. What was the value in 1988? (p. 94)

#27 Write 28.2 million (the number of clerks and salesworkers thought in 1988 to be needed in 1995) as a decimal. (p. 103)

#21 In the first 6 months of 1987, *TV Guide* sold 16,969,260 copies, the most of any magazine. (p. 132)

#38 Some insurance companies give a discount of 15% on homeowner's or renter's insurance if smoke detectors are installed. (p. 144)

In 14 and 15, use the formula N = 7LH. This formula is used by some bricklayers. N is the number of bricks needed in a wall. L is the length of the wall in feet. H is the height of the wall in feet. (p. 165)

Stores use formulas to compute profit. (p. 299)

You have seen the formula p = s – c. (Profit on an item equals its selling price minus the cost of producing it.) (p. 300)

#43 Most business sections of newspapers contain coordinate graphs. Cut out an example of a coordinate graph either from a newspaper or a magazine. (p. 347)

#21 Here are 1987 circulations of four of the best-selling magazines in the United States. Draw a bar graph with this information. *Better Homes and Gardens*, 8,091,751; *The National Geographic*, 10,764,998; *Reader's Digest*, 16,609,847; *TV Guide*, 16,800,441. (p. 370)

Recall that time-and-a-half" means the worker's normal wage is multiplied by $1\frac{1}{2}$ to calculate the overtime wage. (p. 446)

#26 Real estate agents in some places earn 3% of the sale value of a house for helping a person sell the house. This is called the *commission*. (p.458)

#31 Look in a newspaper for the prices of stocks on the New York Stock Exchange. What fractions are used in describing these prices? (p. 477)

The usual unit of typing speed is *words per minute*. (p. 486)

#12 In the book *Big Bucks for Kids*, the author says that a kid can earn $25 for singing 20 minutes at a wedding. At this rate, what should the fee be singing for singing 45 minutes? (p. 509)

#30 In 1986, Volkswagen of America produced about 98,000 cars. In 1987, the company produced 22,000 cars less. So, in 1986 the company produced about __?__ times as many cars as in 1987. (p. 528)

Finding the total cost of a store's stock is a situation leading to a linear expression. (p. 541)

#23 In 198 there were approximately two million, one hundred sixty thousand farms in the United States (p. 554)

#25 In 1930 there were about 295% as many farms as in 1988. About how many farms were there in the U.S. in 1930? (p. 555)

Example The top six wheat producing countries in 1985 were the U.S.S.R., the U.S., China, India, Canada, and France. (p. 605)

Table of 1985 Leading wheat producers. (p. 605)

Calendar

#9 Time: tomorrow, today, yesterday. (p. 24)

#8 In what century were the names commutative and associative first used? (Remember that the years 1901-2000 are the 20th century.) (p. 219)

#12 The days Monday through Friday are weekdays. Saturday and Sunday are on the weekend. A tornado hits a town. (p. 425)

#14 There was no year 0. (p. 582)

Chemistry

The mass of one atom of hydrogen, the lightest element, has been found to be 0.00000 00000 00000 00000 00016 75 gram. (p. 87)

#8 0.00000 00000 00000 00000 0396 gram, the weight of one atom of uranium. (p. 88)
#12 $2.82 \cdot 10^{-11}$ centimeter, the radius of an electron. (p. 89)

Consumer Education

#35 On every cereal box, the amount of protein per serving is listed. (p. 116)
Sugar cubes, number cubes, and dice are examples. (p. 130)
Soft drinks are often sold in 2-liter bottles. (p. 131)
#28 The percent p of discount on an item can be calculated by using the formula $p = 100(1 - n/g)$. In this formula, g is the original price, and n is the new price. (p. 201)
#18 You can buy stickers for digits from 0 through 9 at most hardware stores. They can be used for house addresses and room numbers. (p. 245)
However, in phone numbers, like 555-1212, the dash has no meaning other than to separate the number to make it easier to remember. (p. 248)
#21 Bar graph of the ups and downs of travel costs. (p. 366)
#4 A Thanksgiving meal in 1988 cost the typical person about 45¢ for green beans, 75¢ for a beverage, 30¢ for cranberries, 18¢ for sweet potatoes, and 99¢ for turkey. Put this information into a bar graph. (p. 377)
For 19 and 20, the calories in 1-cup servings of some vegetables are: green beans, 35; cauliflower, 31; lettuce, 5; peas, 110. (p. 380)
The cereal box at the left is an example of a **rectangular solid**. (p. 389)
#27 A stick of margarine is approximately 11.5 cm long, 3 cm wide, and 3 cm high. To wrap the margarine in aluminum foil at least what area of foil is needed? (p. 432)
#34 Mr. Jones tithes. That is, he gives one-tenth of what he makes to charity. If he makes $500 a *week*, how much does he give every *year* to charity? (p. 433)
#22 In August of 1985, some major car dealers were offering 7.7% financing on a new car. What simple fraction is this? (p. 443)
French fries have about 11 calories apiece. (p. 473)
In 9-13, use this information. A hamburger has about 80 calories per ounce. A bun has about 180 calories. A typical french fry has about 11 calories. (p. 476)
#12 There are 12 eggs in a dozen. 100 eggs is how many dozen? (p. 523)
An ounce of hamburger has about 80 calories. (p. 541)
Recall that a typical french fry has about 11 calories. (p. 541)
#15 Use the information from this lesson. A hamburger bun has about 200 calories. (p. 544)
#16 At many schools, a grade of C is worth 2 points, B is worth 3 points, and A 4 points. (p. 544)
#12 A hamburger has about 80 calories per ounce. A bun has about 180 calories. A typical french fry has about 11 calories. If you eat H 4-oz hamburgers with buns, and F french fries, how many total calories will you have eaten? (p. 563)
#19 A tablespoon of butter has about 100 calories. A piece of white bread has about 70 calories. How many calories are there in T tablespoons of butter and P pieces of white bread? (p. 565)
#16 Lisa read that there were about 10 calories per square inch of a 12-inch diameter cheese pizza. if she ate $\frac{1}{8}$ of a pizza, about how many calories would she consume? (p. 603)

Demographics

The census estimate of the U.S. population on July 1, 1980, was 226,545,805 people. (p. 4)

#27 According to these 1980 census data, populations of Salinas – Monterey, CA: 290,444; Appleton – Oshkosh, WI: 291,325; McAllen – Edinburg, TX: 283,229; Pensacola, FL: 289,782. (p. 7)
picture of Monterey, California. (p. 7)
#41 Based on population, which area will get the most federal aid? Kansas City, MO-KS: 1,327,000; Seattle – Everett, WA: 1,606,765: Miami, FL: 1,625,979; Denver, CO: 1,619,921. (p. 13)
One estimate of the world population in the year 2000 is 7 billion. (p. 14)
For instance, in 1970 there were about 3,256,000 births in the United States. In 1977 there were about 3,326,000 births. (p. 26)
Graph of world population 1950-1986. (p. 57)
#26 A census report of August 30, 1988 said the world had grown by 1.8 billion people since 1960. Write 1.8 billion as a decimal. (p. 61)
Example 2 The population of the world passed 5.1 billion in 1989. (p. 63)
#27 Write the three underlined numbers as decimals. The number of students enrolled in grades 9-12 grew from nine million six hundred ninety thousand in 1960 to fourteen million six hundred seventy thousand in 1980. An estimate for 1990 is twelve and a half million. (p. 69)
In 14 and 15, take the U.S. population to be about 230,000,000. (p. 77)
#15 The U.S. population is now increasing at the rate of about 1.6% a year. How many people is this? (p. 77)
#22 The population of Los Angeles, CA was about 100,000 in 1900 and increased 1800% from 1900 to 1950. How many people is that increase? (p. 77)
#4 The probability of a single birth being a boy is about .52 . Convert this number to a percent. (p. 81)
#9 About 1/4 of all families with two children are likely to have two girls. What percent of families is this? (p. 81)
#26 a 105,000,000 the estimated population of Nigeria in 1986. (p. 107)
#29 f If a heart beats 70 times a minute, how many times will it beat in 78 years, the average lifetime of a woman in the U.S.? (p. 107)
#28 In 1987, the world population reached 5.026 billion. (p. 175)
Urban areas are cities and suburbs. (p. 187)
Table of 1970 U.S. rural and urban populations. (p. 187)
Every place in the U.S. is classified either as urban or rural. (p. 187)
In 1970, the population of the New England region was 11,847,186 and the population of the Middle Atlantic region was 37,203,339. (p. 188)
#28 Here are changes in population for the six larges cities in the United States from 1970 to 1980. New York: -823,924; Chicago: -364,185; Los Angeles: 155,049; Philadelphia: -261,786; Houston: 371,603; Detroit: -310,724 . (p. 246)
#29 Of the 5.1 billion people on earth, about 1% live in Egypt. How many people is this? (p. 246)
Table of a state-by-state breakdown of the estimated July 1, 1987, population of each state and its gain or loss since 1980 through migration. (p. 279)
#18 According to the National Center for Health Statistics in 1981, on the average, of 100,000 people born in the years 1901–1906, 57,044 were still alive. (p. 283)
#21 In 1987, the population of Arizona was 437,000 more than in 1980. Suppose the 1987 population was x and the 1980 population was y. How are x, y, and 437,000 related? (p. 293)
#23 Here are census figures for Portland, OR. 1950: 373,628; 1960: 373,676; 1970: 379,967; 1980: 366,383. (p. 293)
#29 Of 100,000 people born in the U.S. in the years 1971–1976, on the average, 98,560 were alive in 1981. (p. 294)
#31 Here are census figures for Milwaukee, Wisconsin. 1950: 637,392; 1960: 741,324; 1970: 717,372; 1980: 636,212. (p. 321)

According to the 1980 U.S. Census, about 3.4 million people of Asian or Pacific Islander ancestry were living in the U.S. in 1980. This number includes 812,000 Chinese; 782,000 Fillipinos; 716,000 Japanese; 357,000 Koreans; 245,000 Vietnamese; 230,000 Pacific Islanders; and 140,000 of other nationalities. (p. 336)

Bar graph titled Number of people of Asian or Pacific Island ancestry in the U.S. in 1980. (p. 337)

Table of 1970 and 1984 population of the six largest U.S. cities. (p. 337)

Bar graph of 1970 and 1984 population of the six largest U.S. cities. (p. 337)

#31 As Example 2 shows, New York has the largest population of any city in the United States. (p. 340)

In 18 and 19, use the following information. Estimated World Population: 1850, 1.1 billion; 1900, 1.6 billion; 1950, 2.5 billion; 2000, 6.1 billion. (p. 361)

For 21 and 22, use the population of the United States during the first four censuses: 1790, 3.9 million; 1800, 5.3 million; 1810,7.2 million; 1820, 9.6 million. (p. 380)

#26 There were about 2,380,000 high school graduates in the U.S. in 1986. Of these, about 58% went to a 2-year or 4-year college. How many college students is this? (p. 387)

Example 2 About 1% of the population of the United States is 85 or older. If the U.S. population is about 243,000,000 how many people are 85 or older? (p. 414)

#9 About 7.7% of the U.S. population is of Spanish origin. If the population is 240,000,000, about how many people in the U.S. are of Spanish origin? (p. 415)

In 1982, there were 20,000 El Salvadorean refugees living in Honduras and 7000 refugees living in Belize. (p. 483)

#14 According to the *1988 World Almanac*, Hawaii had 2743 medical doctors and a population of 964,691 in 1985. California had 72,089 doctors for a population of 23,667,764. Which state had more doctors *per capita*? (p. 487)

#14 In the twenty years from 1950 to 1970, California's population rose by 9,000,000. (p. 495)

Example 2 Compare the 1986 estimated populations of the United States (245,000,000 people) and Canada (26,000,000 people). (p. 498)

#11 The population of Canton, Ohio (home of the pro football Hall of Fame) is 90 thousand. The population of Canton, China (from which we get the name Cantonese food) is about 3 million. (p. 499)

#23 In 1984, Boston had a population of about 570,000 people and an area of about 47 square miles. About how many people is this per square mile? (p. 528)

Economics

#13 Round 328.35, the average consumer price index in December, 1986, to the nearest whole number. (p. 20)

Example A newspaper report in 1988 listed 6.851 million people as unemployed in August. (p. 55)

Quotes form August 28, 1988 issue of the *Chicago Sun-Times* : "Estimates of the winter wheat crop have been increased to 2.02 billion bushels." "There are 1.35 million dollars in scholarships and grants that go unused each year because parents and students don't know that they exist." "Last year some 88 million people shopped by mail or phone ... sales for the year range from $22.2 billion to $33.6 billion." (p. 56)

#9 1.2 trillion dollars, a ceiling (1985) on the national debt. (p. 65)

#33 In 1983, the president of Brazil's central bank said, "We cannot live with 150% inflation. (p. 72)

#10 The cost of living in 1988 was about $3\frac{1}{3}$ times what it was in 1970. What percent is this? (p. 81)

#28 The U.S. budget deficit in 1986 is listed in *The World Almanac* as $220,698 million. (p. 171)

#55 The average number of 16- to 19-year-olds working in 1960 was about 4.1 million. By 1987 the number had increased to about 6.6 million. About how many more teen-agers were working in 1987 than in 1960? (p. 333)

#20 Annual average unemployment rate in the United States: 1983, 9.6%; 1984, 7.5%; 1985, 7.2%; 1986, 7.0%. (p. 339)

#26 Use the information of Question 20. If there were 100,000,000 people in the U.S. work force in 1985, how many people were unemployed? (p. 340)

#38 In Question 20 of the last lesson you picked an interval for a graph of the average annual unemployment rate in the United States. Here is the information again. Put it into a bar graph. 1983, 9.6%; 1984, 7.5%; 1985, 7.2%; 1986, 7.0%. (p. 346)

Graph of U.S. oil imports. (p. 377)

#32 On the average, costs of items in the U.S. tripled from 1967 to 1985. If an item cost x dollars in 1967, what did it cost in 1985? (p. 433)

#26 You often hear that the cost of living is going up. The government calculates a number called the "consumer price index" (CPI) to measure the cost of living. When the CPI doubles, it means that, on average, prices have doubled. (p. 524)

Circle graph of USA population projections by age. (p. 606)

#40 The table below gives approximations in billions of dollars of the top four areas of 1988 U.S. government expenses. Make a circle graph to display this information. (p. 617)

Entertainment

#31 As of 1988, the best-selling record album of all time worldwide had sold over 39 million copies. (p. 69)

Cartoon of Beetle Bailey. (p. 106)

Cartoon of McGurk's Mob. (p. 241)

An equation can be used to find how many tickets have been sold. See Example 2. (p. 303)

Cartoon of Herman. (p. 312)

Cartoon of Moon Mullins. (p. 322)

#48 A one-hour TV program allows $9\frac{1}{2}$ minutes for commercials. How much time is there for the program itself? (p. 332)

#53 The famous actor Henry Fonda died in 1982 at the age of 77. Knowing only this information, what two years might he have been born? (p. 332)

#32 The "D" in 3-D movies is short for "dimensional." (p. 402)

#35 On the TV show "Let's Make a Deal," there are three doors with prizes. One door has a very valuable prize. What is the chance that a contestant picks the door with the valuable prize twice in a row? (p. 433)

#31 Depending on the number of words it contains, a work of prose is often classified as a short story, a novelette, or a novel. For example, a short story is usually less than 10,000 words. To find the number of words per book, editors rarely count them all, but use a rate factor model. (p. 454)

Environmental Studies

Example 2 According to the *World Almanac*, about 715,194,000 acres of the United States are forest. There are 640 acres in a square mile. How many square miles of the U.S. is forest? (p. 522)

So about 32% (almost 1/3) of the land of the United States is forest. (p. 522)

Games

#38 Write twenty-two and ninety-five hundredths in decimal notation. (This was the winning time in seconds in the 1982 World Rubik's Cube competition.) (p. 29)

#25 In the early 1980s, a puzzle known a Rubik's Cube was popular. The object in this puzzle is to rearrange a cube to its original position. There are 43,252,003,274,489,856,000 possible positions. Write this number in English. (p. 60)

The probability that a sum of S will appear when two normal dice are thrown is $(6 - |S - 7|)/36$ (S must be between 2 and 12 for the formula to work.) Use this formula to calculate the probability that a sum of 3 will appear. (p. 471)

Geography, Geoscience

Example 1 The shore of the Dead Sea in Israel, the lowest land on Earth, is 1286 feet below sea level. (p. 23)

#28 b The deepest lake in the world, Lake Baykal in Siberia, has a point that is fourteen hundred eighty-five meters below sea level. (p. 61)

#6 3,500,000 square miles, the approximate land area of the U.S. (p. 65)

#23 The highest mountain in the world is Mt. Everest in the Himalayas. Its peak is 29,028 feet above sea level and is on the Tibet–Nepal border (p. 78)

#26 3,320,000 square miles, the approximate area of the Sahara desert. (p. 82)

#32 In many places in the Midwest, a township is a square 6 miles on a side. What is the area of a township? (p. 161)

Example 2 The highest point in California is the peak of Mount Whitney, 14,494 feet above sea level. The lowest point in California is in Death Valley and is 282 feet below sea level. What is the difference in the elevations of these two points? (p. 290)

#7 The highest elevation in Louisiana is Driskill Mountain, at 535 feet above sea level. The lowest elevation is in New Orleans, at 5 feet below sea level. What is the difference in elevation between these two points? (p. 292)

Picture of the French Quarter, New Orleans. (p. 292)

#22 Here are the number of miles of coastline of those states bordering on the Gulf of Mexico: Alabama, 53; Florida, 770; Louisiana, 397; Mississippi, 44; Texas, 367. (p. 339)

#15 Given are the latitude and longitude of five European cities. (p. 357)

#30 A map (like a roadmap or a map of the U.S.) is a drawing of a contraction of the world. The *scale* of the map is the size change factor. (p. 417)

#24 About 2/3 of the Earth's surface is covered by water. (Some people think we should call our planet Water, not Earth). If a meteor were to hit the earth, what are the chances it would hit on land on a Sunday? (p. 463)

El Paso, Texas, is nicknamed "Sun City" (p. 487)

#10 Below is a map of the region around Santa Fe, New Mexico. (p. 519)

#6 The area of the state of Illinois is about 58,000 square miles. How many acres is this? (p. 523)

#14 The shape of Egypt roughly approximates a trapezoid. The north edge of Egypt is about 900 km long, the south edge about 1100 km long, and the height is about 1100 km. What is the approximate area of Egypt? (p. 592)

#8 The equator of the earth is a circle whose diameter is about 7920 miles. What is the distance around the earth at its equator? (p. 596)

#18. The surface area of the world is given in this lesson.Only about 29.4% of the surface area is land. (p. 612)

#18 b The area of the United States is approximately 3,540,000 square miles. What percent of the total land area of the earth is in the United States? (p. 612)

History of Mathematics

Greeks and Romans used letters for numbers. . . . Europeans most often wrote numbers using Roman numerals. (p. 3)

In the years 600 – 900 A.D., the Hindus developed decimal system. . . . The Arabs wrote about this system. Europeans did not learn about it until 1202 A.D., when Leonardo of Pisa, an Italian mathematician also known as Fibonacci, translated an Arabic manuscript into Latin. (p. 3)

Chart of the development of Arabic Numerals. (p. 3)

Notice that the symbols we use today did not all appear until the late 1400s. (p. 4)

Decimal places first extended to right in 1585 by Simon Stevin, a Flemish mathematician, in 1585. (p. 9)

#39 The inequality signs < and > were first used by Thomas Harriot, an English mathematician, in 1631. Find out something else about this person. (p. 29)

The fraction bar was first used by the Arabs and later by Fibonacci, but it was not widely used until the 1500s. A curved slash, a⁄b was first used by the Mexican Manuel Antonio Valdes in 1784. In the 1800s this developed into the slash in a/b. (p. 34)

Historically we use 10 as a base because we have ten fingers. (p. 59)

In 1650 the symbol $\frac{o}{o}$ was used for percent. (p. 71)

The $\angle$ symbol for angle was first used by William Oughtred in 1657. (p. 117)

Over 2500 years ago, the Babylonians wrote numbers in a system based on the number 60. (p. 118)

The Babylonians divided a circle into 360 equally spaced units, which we call degrees. (p. 118)

#30 Before exponents appeared as raised decimal numbers, some people wrote them as Roman numerals on the same line as the base. In this notation, 2 to the fifth power would be 2V. Modern exponent symbols first appeared in 1637 in a book by René Descartes. Find out something else about this famous mathematician and philosopher. (p. 129)

Table of symbols for the operations of arithmetic and algebra. (p. 139)

Rules for order of operations have been developed only in this century but they are now used world-wide (p. 141)

Algebra is generally considered to have begun in 1591. (p. 146)

The general pattern was first given a name by François Servois in 1814. He used the French word "commutatif", which means "switchable". The English name is the *commutative property*. (p. 216)

For this reason, in 1835, the Irish mathematician Sir William Rowan Hamilton called the general pattern the *associative property*. (p. 216)

Quote from George Polya (1887–1985). (p. 241)

George Polya was a mathematician at Stanford University who was famous for his writing about solving problems. (p. 243)

In a famous book called *How to Solve It*, Polya described what good problem solvers do. (p. 243)

In the Middle Ages, students used a shortcut called the Rule of Three to solve proportions. (p. 511)

In the Middle Ages, students who wanted to solve proportions used a shortcut called the Rule of Three to solve proportions. (p. 512)

A poem written in the late Middle Ages indicates the confusion.

Multiplication is vexation,
Division is bad,
The Rule of Three
Does puzzle me
And practice drives me mad. (Writer anonymous.) (p. 512)

The idea of the square root was known to the ancient Egyptians. The ancient Greeks discovered that square roots of many numbers cannot be simple fractions. The $\sqrt{}$ sign was invented in 1525 by the German mathematician Christoff Rudolff. (p. 575)

The Pythagorean Theorem is the most famous theorem in all geometry. This theorem was known to the ancient Chinese and ancient Egyptians. But it gets its name from the Greek mathematician Pythagoras. Pythagoras was born about 572 B.C. He and his followers may have been the first people in the Western World to know that this amazing theorem is true for any right triangle. (p. 580)

#24 The Pythagoreans worshipped numbers and loved music. They discovered relationships between the lengths of strings and the musical tones they give. Look in an encyclopedia to find out one of the relationships discovered by the Pythagoreans. (p. 583)

#39 The Greek mathematician Archimedes knew that π was between $3\frac{1}{7}$ and $3\frac{10}{71}$. (Archimedes lived from about 287 B.C. to 212 B.C. and was one of the greatest mathematicians of all time.) (p. 599)

The first circle graphs were done by William Playfair in the late 1700s. (p. 605)

#25 George Polya died in September of 1985. He was born in December of 1887. (p.608)

The formulas for the surface area and volume of a sphere were first discovered by Archimedes. (p. 609)

Industrial Art, Design

The sides of this page form right angles at the corners. (p. 122)

Each long side of this page is perpendicular to each short side. (p. 122)

Tops, figure skaters, gears, the Earth, pinwheels, and phonograph records turn. (p. 212)

#37 Phonograph records can be 33 rpm or 45 rpm. (p. 215)

#32 Rectangles are easy to find. Most windows, chalkboards, and doors are shaped like rectangles. (p. 321)

#15 What is the area of a sheet of 8" by 10.5" notebook paper? Ignore the holes. (p. 462)

Begin with a real object. Take a photograph of it. Draw a picture of it. If it's very big, construct a scale model. If it's very small, magnify it. Any of these things lead to similar figures in which you might want to find lengths of sides. (p. 518)

#13 Typing paper is an 8.5" by 11" rectangle. (p. 582)

Meteorology

#16 In Bakersfield, CA, it rains on about 10% of the days in a year. About how many days is this? (p. 77)

#28 It rains or snows on about 42% of the days of the year in Seattle, Washington. About how many days per year is this? (p. 86)

#32 Write $\frac{2}{1000}$ cm, the diameter of a cloud droplet, as a decimal. (p. 112)

In 24 and 25 use the formula $T = t - \frac{2f}{100}$. This formula estimates the air temperature T at an altitude of f feet when the ground temperature is t. (p. 170)

#52 The formula $F = 1.8C + 32$ relates Fahrenheit (F) and Celsius (C) temperature. If C is 10, what is F? (p. 185)

#54 The formula $C = 0.6n + 4$ estimates the temperature C in degrees Celsius when n is the number of cricket chirps in 15 seconds. If a cricket chirps 25 times in 15 seconds, what is an estimate for the temperature? (p. 185)

#19 The highest temperature ever recorded in the U.S. was 134°F, in Death Valley, California, on July 10, 1913. The lowest temperature ever recorded in the U.S. was -80°F, in Prospect Creek, Alaska, on January 23, 1971. (p. 298)

#26 In Montreal, Quebec, the average normal high temperature in January is -6°C. The average normal low temperature is -15°C. What is the average range in the temperature on a January day in Montreal? (p. 302)

#52 O'Hare airport in Chicago is often 5°F colder than downtown Chicago. Suppose the record low for a day is -13°F and was recorded downtown. What was the possible temperature at O'Hare? (p. 332)

#23 The record high and low Fahrenheit temperatures in selected states are: Alaska, 100° and -80°; California, 134° and -45°; Hawaii, 100° and 12°. (p. 339)

Graph comparing recent winters 1986-1988 in Chicago. (p. 354)

#6 On December 24, 1983, an all-time low temperature of -25° set a record for any day in Chicago. Which of the days listed came closest to that record? (p. 356)

70 centimeters of snow per year is the average snowfall in Chicago. (p. 450)

#26 In Anchorage, Alaska, the following low temperatures were recorded for seven days in a week. -14°, -18°, -16°, -8°, -12°, -15°, -13°. To the nearest tenth, what was the mean low temperature? (p. 528)

#29 In Seattle, Washington, it rains 216 days a year on average. What percent of the time is this? (p. 528)

#13 The formula for converting degrees Fahrenheit F to degrees Celsius C is $C = \frac{5}{9}(F - 32)$. Substitute an F for C in the formula and solve to find out what temperature reads the same in both systems. (p. 553)

Music

#25 a In music, two eighth notes take the same time as a quarter note. What addition of fractions explains this? (p. 197)

#16 Mozart wrote 41 symphonies in a lifetime of only 35 years. At this rate, how many symphonies would he have written, had he lived to 70? (p. 509)

#19 The circle graph below tells about the number of players in a string orchestra. What percent of the orchestra are cellos (pictured below)? (p. 614)

Philosophy of Mathematics

In Europe, the decimal numeral for 7 is sometimes written as shown in the cartoon. (p. 7)

#42 In England, the word *billion* does not always mean the number 1 followed by 9 zeros. (p. 57)

#32 A *googol* is one of the largest numbers that has a name. (p. 61)

#40 Mathematics is a world-wide language. Mathematicians from different countries usually use the same symbols. Name some symbols outside mathematics that are used throughout the world. (p. 144)

Because capital and small letters can mean different things, in algebra you should not change capital letters to small letters, or vice versa. (p. 164)

Graphs can show trends. (p. 355)

Here is a true proportion. $\frac{4}{10} = \frac{6}{15}$ In some places in the world, this proportion is written 4:10 = 6:15. In the United States, the colon sign means "ratio." (p. 511)

Physics, Optics

The photograph shows the tracks of tiny atomic particles traveling through liquid helium. The mass of an atomic particle is a very small number usually written in scientific notation. (p. 53)

#28 c Absolute zero is four hundred fifty-nine and sixty-seven hundredths degrees Fahrenheit below zero (p. 61)

#6 0.00008052 second, the time needed for TV signals to travel 15 miles. (p. 88)

#7 0.28 second, a time needed for sound to travel the length of a football field. (p. 88)

#28 The atomic bomb that exploded on Hiroshima had a force equivalent to 20 kilotons of TNT. How many tons is this? (p. 111)

#29 The most powerful bomb ever exploded (by the Soviet Union in a test in 1961) had a force of 57 megatons of TNT. How many tons is this? (p. 111)

#25 Tungsten wire four ten-thousandths of an inch in diameter is used to make filaments for light bulbs. (p. 125)

Physicists use arrows called *vectors* to add forces. (p. 201)

#35 A rope d inches in diameter can lift a maximum of about w pounds where $w = 5000d(d + 1)$. (p. 205)

#25 The famous scientist, Marie Curie, was born in 1867 and died in 1934. From this information, how old was she when she died? (p. 294)

Volume gives the capacity of containers ranging in size from atoms to objects even larger than the Earth. (p. 393)

#25 In Europe, the number zero is sometimes called the *annihilator*. What is the reason for this name? (p. 467)

#19 Ignoring air resistance, it takes about $0.25\sqrt{s}$ seconds for an object to fall s feet. A dime is dropped from the top of the Sears Tower, 1454 feet above the ground. To the nearest second, about how long will it take the dime to reach the ground? (p. 588)

Politics, Law, History

#52 George Bush received about 53.9% of the votes cast in the 1988 presidential election. About 89,000,000 votes were cast. About how many votes did Bush get? (p. 94)

One article in the Constitution reads: The Congress shall have power . . . to fix the standard of weights and measure. (p. 98)

In 1790, Thomas Jefferson proposed to Congress a measuring system based on the number 10. (p. 98)

#26 The Roman poet, Livy, was born in 59 B.C. and died in 17 A.D. From this information, how old was he when he died? (Watch out again. There was no year 0. The year 1 A.D. followed 1 B.C.) (p. 294)

#25 Booker T. Washington was born in 1856 and died in 1915. How old was he when he died? (p. 301)

picture of Booker Taliaferro Washington and his two sons. (p. 301)

#28 a Which of these famous English monarchs had the longest reign? King Henry II, who reigned from ?154 to 1189; King Henry VIII, 1509–1547; Queen Elizabeth I, 1558–1603; Queen Victoria, 1837–1901. (p. 306)

#28 b The present Queen of England, Elizabeth II, ascended to the throne in 1952. In what year could she become the longest reigning queen? (p. 306)

#21 Popular vote in the Presidential election of 1860: Abraham Lincoln, 1,866,352 votes; Stephen A. Douglas, 1,375,157 votes; John C. Breckinridge, 845,763 votes; John Bell, 589,581 votes. (p. 339)

#25 The Northwest Ordinance of 1787 divided many areas of the central United States into square-shaped townships 6 miles on a side. (p. 387)

#25 The flag of the United States has as many stars as states. How many states did the U.S. have when the flag below was in use? (p. 432)

#6 In the 1984 presidential election, Ronald Reagan got 51% of the votes in Massachusetts. He received 1,297,737 votes there. To the nearest ten thousand, how many people in Massachusetts voted? (p. 448)

#25 In the Georgia primary of March 1988, the Reverend Jesse Jackson received about 248,000 votes. This was about 39% of the total. In all, about how many people voted? (p. 477)

In 12 and 13, use the drawing of the flag of the Caribbean island Bonaire. (p. 571)

#26 The U.S. flag has 7 red stripes and 6 white ones. The stripes all have the same thickness. The upper left corner is blue with 50 white stars. Dimensions of a big flag are shown below. (p. 572)

Religion

#26 The book of the Old Testament called Numbers gets it name because it begins with a census of the adult males of the tribes of Israel. The ancients did not have our numerals. So they wrote out the population in words. ". . . of the tribe of Reuben, were forty and six thousand and five hundred." Write this number as a decimal. (p. 6)

#27 In the Canadian province of Manitoba, 26% of the people belong to the United Church of Christ, 25% are Roman Catholic, 12% are Anglican, 7% Lutheran, 6% Mennonite, and 6% Ukrainian Catholic. (p. 201)

#21 Temples in the Baha'i religion have 9 sides. Suppose you view a Baha'i temple from directly overhead. The outline of the temple will have what shape? (p. 228)

Sports

In early 1984, a U.S. women's record in the 200-meter dash was set by Evelyn Ashford. (p. 8)

#34 In 1988, Florence Griffith-Joyner set a women's world record of 10.49 seconds in the 100-meter dash. If this record was lowered by a tenth of a second, what would the new record be? (p. 12)

#37 In the summer of 1988 Florence Griffith-Joyner ran 200 meters in 21.77 seconds. Is this faster or slower than the 1984 time of Evelyn Ashford? (p. 12)

#29 The men's world record in the mile (as of December 1988) was set by the British runner, Steve Cram, in 1985. It is usually written as 3:46.32 . Translate the number 3:46.32 into English (p. 17)

On TV bowling -12 means "behind by 12 pins." The symbol +12 means "ahead by twelve pins." (p. 22)

#8 Football: losing yardage, gaining yardage, no gain. (p. 24)

#15 The wrestler Andre the Giant, whose height is 7'4", is taller than the basketball player Kareem Abdul-Jabbar, whose height is 7'2". (p. 28)

#18 The Preakness, a famous horse race, is $1\frac{3}{16}$ miles long. Convert tis length to a length in decimals. (p. 40)

#28 a Walter Payton ran for thirteen yards on the first play of the game. (p. 61)

#26 Batting "averages" in baseball are calculated by dividing the number of hits by the number of at-bats and usually rounding to the nearest thousandth. In 1970, Alex Johnson (202 hits in 614 at-bats) and Carl Yastrzemski (186 hits in 566 at-bats) had the top two batting averages in the American League. (p. 66)

#25 Some people believe that if a sports team or individual appears on the cover of *Sports Illustrated*, then the team or individual becomes jinxed and will suffer a decline in performance. Researchers at the University of Southern California examined 271 covers. They found that 57.6% of the cover subjects improved in performance. (p. 103)

#19 How many feet are in 440 yards, a common distance in many school running events? (p. 105)
#16 A baseball diamond is really a square in shape. The distance from home to first is 90 feet. What is the area of the square? (p. 128)
#25 The length of the U.S. Grand Prix race is 322.6 kilometers. About how long is this in miles? (p. 129)
#30 The USSR won about 21% of the 138 medals awarded at the 1988 Winter Olympic Games. This was the most medals awarded to one country during those games. How many medals did the USSR win? (p. 153)
#20 the 1988 Super Bowl Champion Washington Redskins, who had a regular season record of 11 wins and 4 losses. (p. 170)
#21 the 1984 U.S. Olympic Women's basketball team, who won all 6 games they played. (p. 170)
#26 In a *Bill James Baseball Abstract*, the expression $\frac{2(HR \cdot SB)}{HR + SB}$ is called the Power/Speed number. (p. 171)
#34 In a single elimination tournament a team plays until it is beaten. Eight teams are to play a single elimination tournament. How many games must be played? (p. 289)
#27 In the 1988 Summer Olympics, an American diver, Greg Louganis, defeated a Chinese diver, Xiong Ni, by one and fourteen-hundredths points. If Ni's total score was 637.47, what was Louganis's score? (p. 305)
Finding the number of seats or the amount of sod needed for a football field are two tasks which use multiplication. (p. 382)
Example 5 26.2 miles (the approximate length of a marathon) is how many feet? (p. 452)
#21 Suppose the probability of a hole-in-one on a particular golf hole is 1 in 10,000. What would then be the probability of making two holes-in-one in a row? (p. 490)
#15 At the end of the 1987-88 season, hockey player Wayne Gretsky had scored 583 goals in 696 games in his career. At this rate, in what game would he score his 600th goal? (p. 509)
For 12 and 13, a bowling ball is approximately 8.59 inches in diameter. (p. 611)
#17 A basketball is hollow with a radius of about 12 cm. About how much material is needed to make it? (p. 614)

Surveying, Measurement

Some instruments need to be accurate to within millionths of an inch. (That's much less than the thickness of this page.) (p. 9)
Decimals are now more common than fractions for measurements. (p. 9)
#13 Round 5280, the number of feet in a mile,
#14 Round 30.48, the number of centimeters in a foot, as follows. (p. 16)
#16 Round 1.609344 the number of kilometers in a mile, a. up to the next thousandth, and b. down to the preceding thousandth. (p. 16)
#12 Round 2.54, the number of centimeters in an inch, to the nearest tenth. (p. 20)
#26 Carpenters often measure in sixteenths of an inch. (p. 38)
Mixed numbers are common in measurement. (p. 39)
#27 27,878,400 square feet, the number of square feet in a square mile. (p. 82)
#28 525,960 minutes, the number of minutes in a 365-day year (p. 82)
#10 $1 \cdot 10^{-8}$ centimeter, the angstrom. (p. 89)
#55 A piece of paper is about 0.005 inches thick. What is that is scientific notation? (p. 94)
Photograph of body parts as units of measurement. (pp. 96-97)
The first units of length were based on the human body. . . . For instance, a "hand" was the width of a person's palm (p. 97)

According to tradition, the *yard* originally was the distance from the tip of the nose of King Henry I of England (who reigned from 1100 to 1135) to the tips of his fingers. The *foot* is supposedly based on the foot of Charlemagne (who ruled France and neighboring areas from 768 to 814). (p. 97)

With the manufacturing of lenses and clocks, accurate measurement was needed outside of science. (p. 98)

The old "English system" or "British Imperial system" has evolved into the **U.S. system** or the **traditional system of measurement**. (p. 98)

The base unit of length in the metric system is the meter (p. 99)

On the ruler, each centimeter is divided into 10 parts. (Each part is a millimeter.) (p. 99)

On rulers, inches are usually divided into halves, fourths, and eighths. (p. 99)

#2 The heights of horses are sometimes measured in hands. What was the way that the length called a hand was originally determined? (p. 101)

#32 there are many units of length that have specialized uses. Find out something about each of these specialized units of length. a. pica b. ell c. link d. chain. (p. 103)

#33 *Your* "inch" is the length of your thumb from the tip to the joint in the middle. How close is your inch to an actual inch? (p. 103)

#34 Look at the picture that opens this chapter. Find out what the lengths of a. a fathom, and b. a cubit are today. (p. 103)

Table of common units in the customary system of measurement. (p. 104)

#14 2.2 pounds (the approximate number of pounds in a kilogram) to ounces. (p. 105)

#16 In some cities, a block is $\frac{1}{8}$ mile long. Convert this to feet. (p. 105)

#17 In the U.S. system, a *rod* is defined as 5.5 yards. How many feet are in a rod? (p. 105)

#18 In Great Britain one *gross ton* = 2240 pounds. The longest passenger liner ever built, the *Norway* , weighs about 70,200 gross tons. How many pounds is this? (p. 105)

#26 b 0.013837 in., the length of 1 point in typesetting. (p. 107)

#27 All systems of measurement in common use in the world today have the same units for time. 1 hour = 60 minutes, 1 minute = 60 seconds (p. 107)

#30 An old song goes, "I love you, a bushel and a peck, a bushel and a peck and a hug around your neck, a hug around your neck and a barrel and a heap, a barrel and a heap and I'm talking in my sleep about you ..." Three units of capacity of fruits and grains are in the words to the song. What are they and how are they related? (p. 107)

So in the late 1700s a movement arose to design a better measurement system. The system devised is called **the international or metric system of measurement**. (p. 108)

Soft drinks today are often sold in 2-liter bottles (p. 109)

Full table of International or Metric System of Measurement (p. 109)

1 km ≈ 0.62 mi. (p. 109)

A doorknob is about 1 m high. (p. 109)

The diameter of an aspirin is about 1 cm. (p. 109)

The thickness of a dime is about 1 mm. (p. 109)

A quart of milk weighs about 1 kg. (p. 109)

An aspirin weighs about 1 g. (p. 109)

A speck of sawdust weighs about 1 mg. (p. 109)

1 liter ≈ 1.06 qt. (p. 109)

1 teaspoon ≈ 5 mL. (p. 109)

In the United States, science, medicine and photography are almost all metric. (p. 113)

Table of conversion between customary and metric. (p. 113)

The protractor is the most common instrument used for measuring angles. (p. 118)

#15 Recall there are 3 feet in a yard. (p. 128)

An acre is a unit equal to 43,560 square feet. Use this fact in Questions 18 and 19. (p. 129)

#15 b The correct answer to part a is in millionths of a cubic foot. This is too precise for many uses. (p. 132)

#16 Remember that there are 12 inches in a foot. (p. 132)

1 cm^3 of water equals 1 mL of water and weighs 1 gram. (p. 132)
There are two systems of measurement in use today in the United States. (p. 133)
In lakes, perimeter measures shoreline. Area measures fishing room. (p. 399)
#20 An *acre* is a unit of area in the U.S. system. 640 acres = 1 square mile. A farm is measured in acres. (p. 401)
#21 Recall that a liter is a metric unit equal to 1000 cm^3. (p. 401)
1 mile = 5280 feet. (p. 452)
#21 1 meter = 39.37 inches. Then 25 inches is about how many meters? (p. 476)
In 18 and 19 use the fact that 1 meter = 39.37 inches. (p. 479)
#34 Name the two conversion factors from the equality 1 foot = 30.48 cm. (p. 481)
#16 There are 640 acres in a square mile. How many acres are in 10,000 square miles? (p. 526)

Technology

Calculators *are* computers; each key triggers a program. (p. 31)
#17 All calculators have a way of allowing to correct a mistake in an entry. You press a key to replace one entry with another. On your calculator, what is this key called? (p. 32)
#18 All calculators have a way of starting from scratch with a new calculation. How is this done on your calculator? (p. 32)
Different versions of this Chinese abacus are still used in some countries for computation. Its operation is based on the powers of 10. (p. 83)
Computers are able to try things very quickly. Even when there is a large number of possible solutions, a computer may use trial and error. (p. 256)
You have probably seen computer-animated graphics. Animations are often done by beginning with a figure, then transforming it. (p. 363)
Graph of number of personal computers in use worldwide. (p. 381)
#26 By the end of 1988, about 40% of households had VCRs. If a town has H households, about how many VCRs would there be? (p. 439)

Transportation

#8 57.54 million people, the number of passengers handled by Chicago's O'Hare Airport in 1987. (p. 65)
Compartments in the outer shell of this dirigible were filled with hydrogen so that it would float in the heavier air. (p. 87)
#28 The narrowest street in the world is in Great Britain. It has a width of $19\frac{5}{16}$ inches. (p. 90)
#23 The St. Gotthard Tunnel in Switzerland is the longest car tunnel in the world. Its length is 16.4 km. How many miles is this? (p. 115)
#41 In a guide book the distance between Paris and London is given as 872 km. How many miles is this? (p. 136)
#29 It is about 1110 km by air from Paris to Rome. In miles, how far is it? (p. 153)
Example 3 On some interstate highways, a car's speed must be from 45 mph to 55 mph. Graph all possible legal speeds s. (p. 177)
In driving from Chicago to Louisville, the quickest way is to go through Indianapolis. (p. 230)
#1 Driving along the California coast, San Luis Obispo is between San Francisco and Los Angeles. A map shows that San Francisco is 241 miles from San Luis Obispo. Los Angeles is 195 miles from San Luis Obispo. By this route, how far is it from San Francisco to Los Angeles? (p. 232)
#8 Driving along Interstate 55 in Illinois from Collinsville to Joliet you will pass through Litchfield and then through Atlanta. (p. 253)

#25 Interstate 25 runs north and south through Wyoming and Colorado through Denver, Colorado Springs, Pueblo, and Cheyenne. (p. 277)

Here is a sketch of part of a map of Chicago. Drexel and Ellis avenues go north and south from 71st to 72nd streets. Then they bend so that they intersect South Chicago Avenue at right angles. (p. 309)

#22 Entrance ramps onto expressways usually form very small angles with the expressway. This is bad for visibility and is quite dangerous. Why aren't entrance ramps perpendicular to expressways? (p. 311)

This view of the United Airlines connecting terminal at O'Hare Airport, Chicago, suggests that each side is a reflection image of the other. (p. 362)

Early pioneers in aviation used the trial and error method in their quest to learn more about aerodynamics. (p. 436)

#7 At 45 kph (kilometers per hour), it takes __?__ hours to travel 90 km. (p. 523)

#25 Some bicycle wheels are 24" in diameter. How far will the bike go if the wheels turn 10 revolutions? (p. 597)

Vocabulary

The word *percent* comes form the Latin words *per centum*, meaning "through 100." (p. 71)

The abbreviation for inch is *in.* (with the period). The abbreviation for ounce is *oz* without a period. (p. 104)

Translation from one language to another is an important communication skill. (p. 149)

Ambiguous means the expression has more than one meaning. (p. 151)

#22 "Trebled" means "multiplied by three." (p. 152)

#17 Consider the vowels A, E, I, O, U. Make two-letter monograms with these vowels, like AO, UE, II. How many two-letter monograms are there? (p. 245)

If you do not know the meaning of a word, look it up in a dictionary. (p. 247)

To *display* means to show. (p. 335)

The word "image" suggests a mirror. Mirrors are the idea behind *reflections*. (p. 362)

#15 Pictured here is a postage stamp from the Polynesian kingdom of Tonga. (p. 374)

The phrase *per capita* means per person. It comes from the Latin words meaning "per head." (p. 487)

#27 Words often have many meanings. Sometimes those meanings do not agree with each other. One of the meanings of trapezoid is "trapezium." (p. 593)

Appendix D: Cultural Contexts in Dolciani's *Algebra 1*

Architecture, City Planning

The design of the Red Rocks Theater, near Denver, Colorado, required skill in electronics and civil engineering as well as architecture. All of these fields involve an extensive use of mathematics. (p. xii)

Engineers design concert halls to ensure acoustic balance. The photo shows a performance at the Philharmonic Hall in West Berlin, where the audience surrounds a central stage. (p. 322)

Career article on Architecture (p. 457)

Astronomy

#3 Mercury freezes at a temperature of approximately 234 K and boils at approximately 629 K. What is the temperature range, expressed in degrees Celsius, to which a mercury thermometer is limited? (p. 111)

#16 Taurus is the zodiac constellation with the greatest number of stars visible to the naked eye, while Aries has the least number of visible stars. Twenty more stars are visible in Cancer than in Aries. Sagittarius has twice as many visible stars as Cancer, and Taurus has twelve more visible stars than Sagittarius. If the number of visible starts in Aries is ten greater than one third the number of visible stars in Taurus, how many stars are visible in Cancer? (p. 141)

Example If it takes 10 s for a heat shield to fall 445 m to the surface of Venus, what is the acceleration due to gravity on Venus? The acceleration due to gravity on Venus is $8.9\ m/s^2$. (p. 144)

Astronauts first landed on the moon on July 16, 1969. (p. 265)

Career article on Aerospace Engineering. (p. 312)

This great spiral galaxy, named M 31, has twice the mass of our galaxy, the Milky Way. The galaxy M 31 is located in the constellation Andromeda, which is $2.2 \cdot 10^6$ light-years from Earth. (p. 380)

Earth is about 150,000,000 km from the sun. (p. 399)

One of the nearest stars, Alpha Centauri, is about 39,000,000,000,000 km from Earth. (p. 399)

For example, the diameter of a galaxy such as the Milky Way, might be expressed as one thousand exameters, or 1000 Em, and the time needed for the sun to revolve around the Milky Way as ten petaseconds, or 10 Ps. (p. 425)

#3 Express each of the following using the appropriate metric prefix. a. the average distance from the sun to the planet Pluto, 5.9×10^{12} m b. the average time needed for sunlight to travel to Pluto, 20×10^3 s c. the mass of ten million plutonium atoms, 4.0×10^{-15} g. (p. 425)

Earth: mass = 5.98×10^{24} kg; radius = 6.38×10^6 m. Moon: mass = 7.34×10^{22} kg; radius = 1.74×10^6 m. (p. 479)

Career article on Astronomy. (p. 612)

Banking, Money, Taxes

The value of one dime is 10 cents. (p. 36)

The value of one quarter is 25 cents. (p. 36)

Career article on Accounting. (p. 142)

I = prt Simple interest = principal • rate of interest • time. (p. 144)

When money is invested, the rate of interest is expressed as a percent of the amount of money invested. The *simple interest* earned annually can be found by multiplying the *principal* (the amount of money invested) by the *annual interest rate*. This can be expressed as follows: simple annual interest = principal • annual interest rate This formula can be written as i = pr, where i is the simple annual interest, p is the principal, and r is the annual rate of interest. (p. 432)

#15 The interest rate required to yield a given income is inversely proportional to the amount of money invested. Chris receives income from $16,000 that she has invested at an annual interest rate of 8%. How much money should she invest to receive the same income if the annual interest rate increases to 10%. (p. 466)

#12 The interest rate required to yield a given income is inversely proportional to the amount of money invested. Bonnie receives income from $20,000 that she has invested at an annual interest rate of 5.5%. How much money must Carl invest at an annual interest rate of 8% in order to receive the same income as Bonnie? (p. 656)

#7 Property tax varies directly as assessed value. The tax on a certain house assessed at $80,000 is $1400. What is the assessed value of a house in the same community for which the property tax is $1610? (p. 679)

Biology

The photo shows archaeological workers at the site of a subway excavation in Boston. Archaeologists often use string grids such as those shown in order to keep track of the location where objects are uncovered. (p. 214)

Biologists, on the other hand, are able to calculate very *small* measurements with the aid of microscopes. (p. 425)

#2 Which of the following specimens (diameter given in parentheses) could be viewed by using an electron microscope but not by using a light microscope? a. a bacterium (1 μm) b. a virus (100 nm) c. an amoeba (50 μm). (p. 425)

The science of statistics is useful in the study of the distribution of characteristics such as weight and life span among animal populations, such as the school of dolphins shown in the photo. (p. 622)

Business

A merchant often mixes goods of two or more kinds in order to sell a blend at a given price. (p. 196)

Many practical problems in business, science, and industry can be solved using the techniques of *linear programming*. **Linear programming** is a method of solving problems in which a quantity represented by a linear equation, often profit or cost, is to be maximized or minimized subject to conditions expressed by a system of linear inequalities. (p. 302)

Career article on pharmacists. (p. 388).

For example, business people and politicians often use marketing surveys and opinion polls to assess the attitude of the *population* (the customers or voters) by studying a *sample* of it. (p. 645)

Calendar

One year consists of 12 • 1 months, or 12 months. (p. 36)
Two years consist of 12 • 2 months, or 24 months. (p. 36)
In 1752, England and its colonies adopted the *Gregorian calendar*, which is the calendar still in use today. (p. 264)

Chemistry

#6 Mercury melts at a temperature of -38.86°C, but it does not boil until it is at a temperature of 356.86°C. What is the difference between the melting and boiling points of mercury? (p. 86)
#10 The melting point of nitrogen is 14.21°C lower than its boiling point. If its boiling point is -195.79°C, what is its melting point? (p. 87)
A chemist mixes solutions of different strengths of a chemical to obtain a solution of a desired strength. (p. 196)
#15 Eighteen-carat gold contains 18 parts by mass of gold to 6 parts of other metals. Fourteen-carat gold contains 14 parts by mass of gold to 10 parts of other metals. How many kilograms of eighteen-carat gold should be mixed with fourteen-carat gold to obtain 60 kg of an alloy containing 17 parts gold and 7 parts of other metals? (p. 203)
The diameter of a silver atom is about 0.00000000025 m. (p. 399)
Chemists, druggists, and others frequently are confronted with situations where they find it necessary to mix ingredients. (p. 433)
#23 In calcium sulfate the ratio, by mass, of calcium to sulfur is 5 : 4 and of calcium to oxygen is 5 : 8. How many grams of each are there in 34 g of calcium sulfate? (p. 456)
#1 The volume V of a gas at a fixed temperature varies inversely as the pressure P. (p. 464)

Consumer Education

Niacin, Riboflavin, Thiamine, are types of vitamins. (p. 302)
Amount of the discount = Original amount • % discount. (p. 432)
The cost of operating an appliance varies jointly as the number of watts, hours of operation, and the cost per kilowatt-hour. If it costs 9¢ to operate a 1200-watt dishwasher for 40 min at a rate of 5¢ per kilowatt-hour, how much does it cost to operate a 3000-watt air conditioner for 4 h at the same rate? (p. 679)

Demographics

#12 The population of Concord, New Hampshire in 1980 was 400 greater than half the population of Euclid, Ohio. The sum of the two populations was 29,600 less than twice the population of Euclid. What was the population of each city in 1980? (p. 140)
#8 includes a table of immigration to the U.S. in an unknown year (p. 630)
#15 United States mortality tables show that about 95% of people born now can expect to be alive at age 40, but only about 75% of those born now will still be alive at age 65. What is the experimental probability that person born now who reaches age 40 will still be alive at 65? (Assume that life expectancies do not change in the next 65 years.) (p. 652)

Environmental Studies

Ecologists often need to know how many animals of a certain species live in a particular area. (p. 658)

One way to estimate the number of animals is to catch some animals, tag them, and release them. (p. 658)
There are several possible sources of error in this procedure. (p. 658)

Games

Simple games of chance offer ideal situations for understanding the notion of probability. (p. 645)
Picture of outcomes of a die. If the die is not loaded you can assume that each outcome is equally likely. (p. 645)
There are 4 jacks in a standard bridge deck of 52 cards. (p. 646)

Geography, Geoscience

Career article on Oceanography. (p. 16)
Example Mount McKinley, Alaska, is 6194 m above sea level, and Death Valley, California, is 86 m below sea level. Mount McKinley is 6280 m higher than Death Valley. Death Valley is 6280 m lower than Mount McKinley. (p. 86)
#9 Mount Everest rises 8848 m above sea level, which is 19,763 m higher than the floor of the Marianas Trench in the Pacific Ocean. What is the depth of the Marianas Trench? (p. 87)
#11 There is a difference of approximately 52° latitude between point Barrow, Alaska and Ka Lae, Hawaii. If Ka Lae is located at 19° north latitude, what is the latitude of Point Barrow? (p. 87)
#11 The Rhine River is 160 km shorter than the St. Lawrence River. The Amazon River is 800 km longer than five times the length of the Rhine River. The Zambezi River is twice as long as the St. Lawrence River. How long is the Amazon if it is 1440 km longer than the sum of the lengths of the other three rivers? (p. 140)
#13 The Panama Canal is 2 km shorter than twice the length of the Suez Canal. The sum of the lengths of the two canals is 121 km greater than three fourths the length of the Panama Canal. Find the length of each canal. (p. 140)
To describe the steepness, or slope, of a hill, you may estimate the amount of vertical *rise* of the hill that corresponds to a certain amount of horizontal *run*, then calculate the ratio of the rise to the run. (p. 242)
#7 includes a table of river lengths in kilometers. (p. 630)
#4 The St. Lawrence River is 120 km longer than 3 times the length of the Albany River. The Yukon River is 130 km longer than the St. Lawrence River. The combined lengths of the Albany River and the St. Lawrence River is 850 km greater than the length of the Yukon River. How long is the Yukon River? (p. 676)

History of Mathematics

Venn diagrams are named after John Venn, an English mathematician who was among the first to use them extensively. (p. 8)
Biography of Marjorie Lee Browne (1914-1979). (p. 10)
Biography of Karl Friedrich Gauss (1777-1855). (p. 61)
#3 The German mathematician Emmy Noether was born in 1882 and died after her birthday in 1935. What was her age in years when she died? (p. 86)
#4 If the Greek mathematician Pythagoras was born in 572 B.C. and died after his birthday in 497 B.C., what was his age in years when he died? (p. 86)
The Cartesian coordinate system in named after René Descartes, a seventeenth century French mathematician and philosopher who developed the early ideas about the system. (p. 217)
Biography of Ada Lovelace (1815-1852). (p. 240)

Biography of François Viète (1540-1603). (p. 348)
Biography of Julio Rey Pastor (1888-1962). (p. 396)
Biography of Charlotte Angas Scott (1858-1931). (p. 436)
The development of this theorem is credited to Pythagoras, a famous Greek philosopher and mathematician who lived from about 580-500 B.C. It is believed that his method of proof was based on comparison of areas. (p. 505)
Biography of Chu Shih-chieh (13-14th centuries). (p. 556)
Biography of Norbert Rillieux (1806-1894). (p. 607)

Industrial Art, Design

An astronaut aboard the space shuttle Challenger rides the remote manipulator system (RMS) arm during a space walk. The arm is controlled by another astronaut from within the cabin and may be used to retrieve and to release satellites and to shift equipment within the cargo bay. (p. 112)
Pictured below is a device that is called a beam balance. (p. 113)
These wind turbines, located in Tehachapi Pass, California, are using wind currents to generate electricity. Using natural sources of power helps in the conservation of energy. (p. 266)
Robots are often used in manufacturing and industry to increase the speed and quality of production. The photo shows robots welding a car chassis in a St. Louis assembly plant. (p. 426)
This suspension bridge in Portland, Maine, was completed in 1866, and its original iron cables are still in place. Each of the cables is shaped like part of a parabola. (p. 542)
Modern roads and buildings such as those pictured in this aerial view of Boston, are triumphs of civil engineering. This branch of engineering depends heavily on trigonometry. (p. 578)

Meteorology

The icicles hanging from the trees in this photo were formed when the air temperature dropped to 0°C, or below. The body of water in the photo has maintained a higher temperature, however, and has remained unfrozen. Recorded temperatures in the United States have ranged from -62°C to 57°C. (p. 48)
#5 The highest temperature recorded in the world is 58°C, while the lowest temperature recorded is -88.3°C. What is the difference between these record high and low temperatures? (p. 86)
#7 When the wind is blowing at 72 km/h, an actual temperature of 2°C can feel like a temperature of -17°C. What is the difference between these temperatures? (p. 86)
#8 An actual temperature of 32°C can feel like a temperature of 50°C when the relative humidity is 90%. What is the difference between these temperatures? (p. 86)
Temperature can be measured with a thermometer. A common type of thermometer is one that contains a liquid, usually mercury or alcohol, in a narrow glass tube. (p. 110)
Anders Celsius, a Swedish scientist who lived from 1701 to 1744, devised a temperature scale that today bears his name. (p. 110)
A few years before the development of the Celsius thermometer, a German instrument maker, Gabriel Daniel Fahrenheit, designed a thermometer using a different scale (p. 110)
Another important temperature scale is the *absolute*, or *Kelvin*, scale. (p. 111)
#4 The formula $C = \frac{5}{9}(F - 32)$ gives the temperature C in degrees Celsius in terms of the temperature F in degrees Fahrenheit (p. 147)
#4 In one day the temperature in Browning, Montana, dropped from 7°C to -31°C. How many degrees did the temperature drop? (p. 676)

Philosophy of Mathematics

A **formula** is an equation that states a relationship among quantities represent by variables. (p. 144)

There are many everyday terms that have a special meaning in mathematics. Sometimes the everyday use of such terms helps in understanding their use in mathematics. (p. 242)

Scientific notation provides an efficient way to read and write and the very large and very small numbers that frequently occur in scientific measurements. (p. 399)

Scientific notation is also useful in some numerical computations, since numbers written in scientific notation can be multiplied and divided easily using the laws of exponents. (p. 399)

Percents are often used to describe changes in quantities. (p. 431)

Statistics is the science of organizing and analyzing a set of numerical facts, or *data*, so that probable conclusions can be drawn from the data. (p. 623)

Physics, Optics

D=rt: Distance traveled = rate • time traveled. (p. 144)

The formula $x = \frac{1}{2}at^2$ gives the distance x in meters that is traveled in t seconds by a freely falling object near the surface of a planet where the acceleration due to gravity is a m/s^2 (meters per second squared). (p. 144)

When a light ray strikes a reflecting surface, the measure, i, of the *angle of incidence* is equal to the measure, r, of the *angle of reflection*. (p. 187)

An object that moves at a constant speed, or *rate*, is said to be in **uniform motion**. A formula that is often used in solving problems involving uniform motion is: rate • time = distance, r • t = d. The formula may also be used for motion that is not uniform if the *average rate* is considered. (p. 189)

A **velocity** specifies the direction of travel as well as the speed. (p. 212)

the **acceleration** of an object can defined as the rate at which its velocity changes over a given period of time. (p. 212)

The simplest case of acceleration to analyze involves an object that is moving in a straight line at a constant acceleration. (p. 212)

Air speed: the speed of the airplane in still air. Wind speed: the speed of the wind. Tail wind: a wind blowing in the same direction as the one in which the airplane is heading . Head wind: a wind blowing in the direction opposite to the one in which the airplane is heading. Ground speed: the speed of the airplane relative to the ground. With a tail wind: ground speed = air speed + wind speed. With a head wind: ground speed = air speed – wind speed. (p. 289)

The height h in meters that an object will reach in t seconds when it is thrown upward from the ground with an initial speed of r meters per second is given by the formula $h = rt - 4.9t^2$. (p. 369)

#1 The frequencies of radio waves range from one tenth megahertz, or 0.1 MHz, to one tenth gigahertz, or 0.1 GHz. Are radio frequencies greater or less than the frequencies of visible light, which range from 0.4 PHz to 0.75 PHz? (p. 425)

In Sections 4-8 and 6-6 you solved problems involving uniform motion using the following formula. distance = rate • time. (p. 446)

The distance that a spring is stretched is directly proportional to the mass of the object that is stretching the spring. (p. 459)

#6 The mass of a uniform copper bar varies directly as its length. If a bar 40 cm long has a mass of approximately 420 g, find the mass of a bar 136 cm long. (p. 461)

#16 A plastic rod changes length in direct proportion to the change in temperature. At 22°C it is 500 mm long. At 42°C it has increased by 0.05 mm. What is the length if the temperature drops to 18°C? (p. 462)

#2 The current I in an electrical circuit of fixed voltage varies inversely as the resistance R. (p. 464)

#4 The frequency f of an electromagnetic wave is inversely proportional to the length of the wave. (p. 464)

#5 The frequency of a radio wave is inversely proportional to the length of the wave. If a wave of length 500 m has a frequency of 600 kHz (kilohertz), find the frequency of a wave whose length is 750 m. (p. 465)

#7 The force needed to pry up a rock varies inversely as the length of the crowbar used. Using a crowbar 100 cm long, a force of 150 N (newtons) is needed to pry up a certain rock. How long a crowbar is needed to pry up the same rock when the force applied is 120 N? (p. 465)

#9 A pulley revolves at a speed that is inversely proportional to its diameter. A pulley with a diameter of 12 cm is belted to a pulley with a diameter of 8 cm. If the smaller pulley is revolving at a rate of 96 rpm (revolutions per minute), how fast is the larger pulley revolving? (p. 465)

Example The pressure required to force water through a pipe varies directly as the square of the speed of the water and inversely as the diameter of the pipe. (p. 469)

#14 If a wire carries an electric current for a given time, the heat developed varies jointly as the resistance and the square of the current. (p. 471)

#16 The heat loss through a window pane varies jointly as the difference of the inside and outside temperatures and the window area, and inversely as the thickness of the pane. (p. 471)

#18 The centrifugal force of an object moving in a circle varies jointly with the object's mass and the radius of the circular path, and inversely with the square of the time it takes to complete one full circle. (p. 471)

#21 The heat generated by a stove element varies directly as the square of the voltage and inversely as the resistance. (p. 471)

#22 The power in an electric circuit varies jointly as the resistance and the the square of the current. (p. 471)

#16 The distance that a free falling body travels varies directly as the square of the time it has been falling. If an object falls 320 m in 8 s, how far will it fall in 3 s? (p. 475)

#15 The frequency of a radio wave is inversely proportional to the wave length. If the frequency is 60 Mc/s (megacycles per second) for a wave 5 m long, what is the frequency for a wave 3 m long? (p. 476)

The force that pulls a javelin back to Earth is the **force of gravity**, or **gravitational force**. (p. 479)

If air resistance is small, all objects fall to Earth at the same rate. (p. 479)

However, because the surface of Earth is curved and light travels in straight lines, from any given height you can see only as far as the *horizon*, where your line of sight just touches Earth's surface. (p. 540)

#38 When a car starts from rest and travels at a constant acceleration, the distance that it travels varies jointly as its acceleration and the square of the time that it has been traveling. (p. 576)

When an airport radar antenna revolves, as shown in Figure 1, you can think of its beam as generating an *angle*. (p. 579)

Politics, Law, History

Table of the first 20 Presidents and their birth dates. (p. 626)

Sports

Quick acceleration is important in motorcycle racing. (p. 156)
Batting average is a ratio of safe hits to times at bat . (p. 649)

Surveying, Measurement

Whenever you count or measure, you use numbers. (p. 1)
You are probably familiar with such metric prefixes as kilo-, centi-, and milli-, but have you ever encountered the prefixes giga-, nano-, or pico-? (p. 425)

Technology

Career article on broadcasting. (p. 188)
#1 the first telephone call: June 3, 1875. (p. 265)
#3 the first commercial television broadcast: July 1, 1941. (p. 265)
It is often useful to be able to solve problems that involve finding how long it takes to accomplish a task when a steady rate of work is assumed. If the *work rate* is the fraction of the whole job that can be done per unit of time, then work rate • time = work done (p. 443)
#15 The number of persons needed to do a job varies directly as the amount of work to be done and inversely as the time in which the job must be done. If 2 people can cut 6 trees into fireplace logs in 8 h, how long will it take 6 people to cut 12 trees into logs? (p. 471)
#56 The number of persons needed to do a job varies directly as the amount of work to be done and inversely as the time in which the job must be done. If two typists can type 210 pages of manuscript in three days, how many typists will be needed to type 700 pages in two days? (p. 538)

Transportation

#2 the first scheduled passenger service using airplanes: August 25, 1919. (p. 265)
#6 The time needed to travel from one place to another is inversely proportional to the speed. A person traveling 72 km/h can go from New Glasgow to Pictou in 10 h. How fast must the person travel to make the trip in 9 h? (p. 465)
The photo shows a hot air balloon fiesta in Albuquerque, New Mexico. From these balloons, passengers are able to see great distances. (p. 480)
Career article of transportation engineering. (p. 503)
Note that a navigator gives the *bearing*, or *heading*, of a vector by measuring *clockwise* the angle from north to the vector. (p. 605)

Vocabulary

The three dots after the 5 are read "and so on" and indicate that the pattern established by the listed numbers continues without end. (p. 11)
A sentence formed by joining two sentences with the word *and* is called a **conjunction** (p. 170)
A sentence formed by joining two sentences with the work *or* is called a **disjunction** (p. 171)
A sentence formed by joining two sentences with the word *and* is called a *conjunction*. For a conjunction to be true, *both* of the joined sentences must be true. A sentence formed by joining two sentences with the work *or* is called a *disjunction*. For a disjunction to be true, *at least one* of the joined sentences must be true. (p. 207)

A prefix is a letter or a group of letters that is placed before a *base word* or a *root* to make another word. (p. 349)
The word **percent** (often denoted by %) means "per 100" or "divided by 100." (p. 431)
Percent means "per 100." (p. 472)
The word *trigonometric* comes from two Greek words: *trigonon* (triangle) and *metron* (measure). (p. 586)

Appendix E: Cultural Contexts in Saxon's *Algebra 1*

Architecture, City Planning

#2 We estimate that the giant pyramid of Cheops near Cairo contains 2,300,000 blocks of stone. If the builders only used 80 percent of the available blocks, how many blocks were available? (p. 195)

Banking, Money, Taxes

#1 There were 143 more Susan B. Anthony dollars than there were quarters. If the total value of the coins was $153, how many dollars were there? (p. 253)

Business

Formulas $\frac{P}{100} \cdot$ base = percentage, and rate $\cdot$ base = percentage are used almost exclusively in the business world. (p. 137)

Chemistry

In science courses, it is sometimes necessary to use extremely large numbers and/or extremely small numbers. For example, to calculate the number of molecules in 1000 liters of a gas, it would be necessary to multiply 1000 times 1000 times a very large number such as 26,890,000,000,000,000,000 which represents the number of molecules in a gas in a cubic centimeter. (p. 216)

Geography, Geoscience

For instance, the circumference of the earth at the equator is 24,874 miles (p. 295)

History of Mathematics

The system of numeration that we use to designate numbers is called the decimal system. It was invented by the Hindus of India, passed to their Arab neighbors, and finally transmitted to Europe circa 1200 A.D. (p. 5)

The ancients did not understand or use negative numbers (p. 6)

In the 1950s the so-call new math appeared, and among other things it introduced the **number line** at the elementary algebra level. (p. 6)

The figure in its entirety is called a system of **Cartesian coordinates** or a **Cartesian coordinate system** after the famous seventeenth-century French philosopher and mathematician René Descartes (p. 151)

While the Pythagorean Theorem was known to the Egyptians as early as the Middle Kingdom (circa 2000 B.C.), the geometric proof of the theorem is normally attributed to a Greek philosopher and mathematician named **Pythagoras**. Pythagoras was born on the Aegean island of Samos and was later associated with a school or brotherhood in town of Crotona on the Italian peninsula in the sixth century B.C. (p. 283)

Algebra books published in the late nineteenth century and early twentieth century did not contain the word *function*. These books spoke of equations: linear equations, quadratic equations, and other types of equations. The calculus books of the 1930s and 1940s used the word function to describe some equations (pp. 325-326)

Industrial Art, Design

RPM (revolutions per minute) varies inversely as the number of gear teeth. If 100 teeth result in 10RPM, what RPM would result if the number of teeth were reduced to 25? (p. 369)

Philosophy of Mathematics

Instructors at the Algebra 1 level and in higher courses usually prefer the improper fraction to the mixed number. (p. 76)

Many people call decimal numbers decimal fractions because terminating decimal numbers can be written as fractions. (p. 85)

Note the use of the words *factor out*. This phrase is meaningful even though some authorities insist that it is redundant and that the single word *factor* will suffice. However, this slight redundancy is not harmful, especially since it is a natural redundancy. (p. 102 footnote)

Simplification of rational expressions by factoring is seldom seen in real life but often in algebra textbooks. These problems are really contrived problems designed to give the reader practice in factoring and canceling. (p. 273 footnote)

It is interesting to not that mathematicians often disagree on definitions and on terminology. Most authors use similar definitions, but not all do. (p. 282)

Mathematicians sometimes use the word *mean* to mean middle and the word *extreme* to mean end. Thus the mean terms in the multiplication shown are the middle terms, and the extreme terms are the end terms. (p. 371 footnote)

Physics, Optics

The distance required for an automobile to stop is directly proportional to the square of its velocity. (p. 375)

The distance a body falls varies directly as the square of the time that it falls. (p. 375)

The weight of a body on or above the surface of the earth varies inversely with the square of the distance from the body to the center of the earth. (p. 376)

Surveying, Measurement

All positive real numbers can be used to describe physical distances when used with descriptive units such as feet, yards, mile, meters, inches, kilometers, centimeters. (p. 6)

Vocabulary

The Latin word for exchange is *commutare*, and in later algebra courses this definition will be restricted to two numbers and called the commutative property. (p. 16 footnote)

trichotomy, from the Greek *trikha* meaning "in three parts." (p. 112 footnote)

Centum is the Latin word for 100, and thus percent literally mans "by the 100." We often use the symbol % to represent the word percent. (p. 136)

The words **finite** and **infinite** are basic words and are difficult to define. The word *finite* implies the thought of bounded or limited, while the word *infinite* implies the thought of unbounded or without limit. (p. 173)

The Greeks must have drawn some of their right triangles as this one is drawn because the word *hypotenuse* comes from the Greek words *hupo* meaning "under" and *teinein* meaning "to stretch," so hypotenuse means "stretched under." (p. 285 footnote)

Binary operation: *Bi* from the Latin word *bis* meaning "two" as in bicycle. (p. 393 footnote)

Appendix F: Cultural Contexts in *UCSMP Algebra*

Architecture, City Planning

#17 The Sears Tower in Chicago is about 443 meters tall. If a person two meters tall is 100 meters from the front door, about what is the measure of the angle needed to see the top? (p. 664)

Astronomy

#34 The radius of the earth is about 6.37 million meters. Write this quantity in the following ways. (p. 67)

Sirius, the brightest star in the night-time sky, is 8.7 light years form Earth. Procyon, a bright star near Sirius in the sky, is 11.3 light light years from Earth. Let m be the distance between Sirius and Procyon. What values can m have? (p. 98)

Example 4 The Transamerica Building in San Francisco has the shape of a pyramid. Each floor is approximately square. (p. 115)

#13 In New York City, the sun's rays make a $72\frac{1}{2}°$ angle with the ground at noon on June 21, the first day of summer (about June 21). On the first day of winter (about December 21), the angle is $25\frac{1}{2}°$. By how much do the angles differ? (p. 116)

The range of temperatures recorded on the planets of our solar system is 682°C. The minimum temperature of -220°C was recorded on Pluto. The maximum temperature was recorded on Venus. (p. 118)

The maximum recorded temperature is 462°C. (p. 118)

#2 The range of temperatures recorded on Mercury in one day was 797°F. The minimum temperature recorded that day was -23°F. (p. 120)

Our galaxy looks somewhat like the Andromeda Galaxy pictured here. (p. 155)

A formula for computing the number of planets N currently supporting intelligent life is $N = T \cdot P \cdot E \cdot L \cdot I \cdot C \cdot A$ where T = *Total* number of stars in our galaxy, P = fraction of stars with a *Planetary* system, E = fraction of planetary systems with a planet that has an *Ecology* able to sustain life, L = fraction of planets able to sustain life on which *Life* actually develops, I = fraction of planets with life on which that life is *Intelligent*, C = fraction of planets with intelligent life on which that life could *Communicate* outwardly, A = fraction of planets with communicating life which is *Alive* now. (p. 155)

In *Space*, a novel by James Michener, values given for these variables are $T = 400{,}000{,}000{,}000$, $P = \frac{1}{4}$, $E = \frac{1}{2}$, $L = \frac{9}{10}$, $I = \frac{1}{10}$, $C = \frac{1}{3}$, and $A = \frac{1}{100000000}$. (p. 155)

#20 other than the sun, the star nearest to us, Alpha Centauri, is about $6.2 \cdot 10^{12}$ miles away. Earth's moon is about $2.4 \cdot 10^5$ miles from us. If it tock astronauts about 3 days to get to the moon in 1969, at that speed how long would it take to get to Alpha Centauri? (p. 456)

Example 2 The radius of the Earth is approximately $6.36 \cdot 10^3$ km. (p. 459)

The volume of the Earth is approximately $1.08 \cdot 10^{12}$ km^3. (p. 459)

#5 The radius of Earth's moon is approximately $(1.773 \cdot 10^3)$ km. Calculate its approximate volume. (p. 461)

#28 The radius of Jupiter is 11 times that of Earth. The volume of Jupiter is how many times the volume of Earth? (p. 462)

#20 The radius of the sun (roughly a sphere of gas) is about $4.33 \cdot 10^5$ miles. Estimate the volume of the sun. (p. 470)

#54 The moon is nearly a sphere with radius $1.05 \cdot 10^3$ miles. (p. 473)
A golf ball was hit on the moon. (p. 634)
For example, the top row tells that Mercury is about ten billion meters from the sun. (p. 682)
#10 5,020,000,000,000,000,000,000,000,000 tons, the mass of Sirius, the brightest star. (p. 686)

Banking, Money, Taxes

#16 The formula $c = 20(n - 1) + 25$ gives the cost of first-class postage in 1988. In the formula, c is the cost in cents and n is the weight for the mail rounded up to the nearest ounce. What does it cost to mail a 3.2 ounce letter? (p. 49)
Many situations use both positive and negative numbers. You may add gains and losses in a football game, deposits and withdrawals in savings accounts, profits and losses in business endeavors, and so on. (p. 62)
#29 In the summer of 1987, an American dollar was worth about .62 English pounds and one dollar would buy about 6.1 French francs. About how many francs could be bought for 75 pounds? (p. 220)
#57 For every dollar, you could get 2.03 Deutsche Marks (the currency in West Germany) in 1986. If a crystal vase cost 120 deutsche marks in 1986, what would the cost be in dollars? (p. 258)
Graph showing the cost (in cents) of sending a one-ounce first class letter. (p. 375)
Example 3 In 1988, postage rates for first-class mail were raised to $.25 for the first ounce and $.20 for each additional ounce. Graph the relation between weight and cost for whole number weights. (p. 378)
Graph of postage rates. (p. 378)
An equation for the cost C to mail a first-class letter weighing w whole ounces is $C = .25 + .20(w - 1)$. (p. 378)
#6 In 1987, postal rates were 22¢ for the first ounce and 17¢ for each additional ounce. Graph the relation between weight and cost for whole number weights. (p. 379)
Banks, savings and loan associations, and credit unions will pay you to give them your money to save. (p. 424)
Interest is always a percent of the principal. The percentage the money earns per year is called the **annual yield.** (p. 424)
Savings accounts pay **compound interest**, which means that the interest earns interest. (p. 425)
#12 A bank advertises an annual yield of 6.8% on a 6-year CD (certificate of deposit). If the CD's original amount was $2000, how much will it be worth after 6 years? (p. 427)
Often yields for accounts are in newspaper ads. (p. 428)
Example 3 In 1985, there were approximately 3.6 billion one-dollar bills in circulation and about 227 million people in the U.S. How many dollar bills was this per person? (p. 454)
#3 In 1985, there were approximately 929 million five-dollar bills in circulation and 227 million people in the U.S. Convert these words to powers of 10 and find the number of five-dollar bills per person. (p. 455)
The largest money matters most adults deal with are salary or wages, payments on loans for cars, trips, etc., insurance, social security payments and benefits, and home mortgages or rent. (p. 476)
Each of these items involves paying or receiving money each month or every few months or every year. (p. 476)
#24 It was reported in *USA Today* on July 12, 1985 that the Federal National Mortgage Association's first quarter earnings increased 363% to 11.1 million in the second quarter. (p. 507)

Biology

In 14 and 15, an adult's normal weight w (in pounds) can be estimated by the formula $w = \frac{11}{2}h - 220$ when his or her height h (in inches) is known. (p. 28)

#18 b Heights of adult *Tyrannosaurus rex* skeletons range from about 12 m to 15 m. (p. 34)

#36 Collections of animals frequently are given special names. Match the group name with the correct animal as in "school of fish." (p. 35)

#14 Neanderthal man lived in Europe, North Africa and Central Asia from about 200,000 B.C. to about 30,000 B.C. How long was this? (p. 116)

The estimated probability of being able to roll your tongue is $\frac{1}{8}$. The estimated probability of having attached earlobes is $\frac{1}{16}$. What is the probability of a person being able to roll his or her tongue and having attached earlobes? (p. 204)

Being overweight increases the probability that a person will suffer from heart disease. (p. 221)

Chart to test risk of heart disease. (p. 221)

Example 3 The Illinois Department of Public Health reported that for the years 1971 to 1982, 4865 skunks were examined for rabies. Of that number 2162 actually had rabies. What percent of the skunks tested actually had rabies? (p. 222)

Example 4 Life expectancy has increased from an estimated 18 years in 3000 B.C. to 73.8 years in 1980. Using a ratio, compare these like expectancies. (p. 222)

#5 The Illinois Department of Health reported that for the years 1971 to 1982, of 230 horses tested for rabies only 16 actually had the disease. What percentage of horses tested had rabies? (p. 223)

#9 A girl born today in the United States has a life expectancy of about 79 years. A boy has a life expectancy of about 72 years. (p. 224)

In fact, in almost the entire world, 37° is the value used for normal body temperature, not 98.6°, because the U.S. is the only major country that still measures temperature using the Fahrenheit scale. (p. 268)

In 13 and 14 use this information. Lengths of human bones are related by linear sentences. These can be used to estimate heights of individuals if certain bones are found. Archaeologists, paleontologists, and forensic scientists all use these techniques. (p. 270)

#24 If the height h of a man and the length f of his femur (both in centimeters) are approximately related by the formula $h = 69.089 + 2.238f$, what should be the length of a femur of a 6-ft man? (p. 283)

#24 In the U.S., the expectancy L of a 12- to 16-year-old white male is given by the formula $L - 60.7 = .95(A - 12)$ where A is the age of the person. What is the life expectancy of a 14-year-old white male according to this formula? (p. 307)

#57 If the height h of a woman and the length f of her femur (in cm) are related by the formula $h = 61.412 + 2.317f$, estimate the length of the femur of a woman 150 cm tall. (p. 310)

#15 A human hair is 0.1 mm thick. Under a microscope it appears 15 mm thick. How many times is it magnified? (p. 240)

Older people tend to lose height. (p. 369)

#12 It was reported in 1960 that the total length y and the tail length x of females of the snake species lampropeltis polyzina have close to a linear relationship. When $x = 60$ mm, $y = 455$ mm. When $x = 140$ mm, $y = 1050$ mm. (p. 395)

In 25 and 26, use the average weights (in kg) for girls between birth and age 14, as given below. (p. 419)

Rabbits are not native to Australia , and in 1859, 22 rabbits were imported from Europe as a new source of food. Australia's conditions were ideal for rabbits and they flourished. (p. 429)

#24 Exponential decay gets its name from the decay of elements in nature, called radioactive decay. Radioactive decay is used to approximate the age of archaeological objects that were once alive. This can be done because all living things contain radioactive $carbon_{14}$, which as a half-life of 5600 years. (p. 437)

$Carbon_{14}$ dating gives the age of this Tyrannosaurus rex fossil at about 70 million years. (p. 437)

#24 You are descended from two natural parents each of whom had two natural parents, each of whom had two natural parents, and so on. (p. 447)

#34 The average 8th grader has a volume of about 3 cubic feet . (p. 457)

#11 0.0009 meters, the approximate width of a human hair. (p. 686)

Business

#31 Shoe size s and foot length F (in inches) for men are related by the formula $s = 3F - 24$. What is the show size of someone whose foot is $10\frac{1}{2}$ inches long? (p. 35)

#18 The value of IBM stock went down $2\frac{3}{8}$ points on July 30. On July 31 and August 1 it went up $\frac{3}{8}$ of a point each day. Find the net change in IBM stock over this three-day period. (p. 79)

#27 To estimate the number of bricks N needed in a wall some bricklayers use the formula $N = 7LH$ where L and H are the length and height of the wall in feet. About how many bricks would a bricklayer need for a wall 8.5 feet high and 24.5 feet long? (p. 79)

Example 3 During the week of June 10, 1985, the price of IBM stock changed as follows: Monday, up $\frac{1}{4}$; Tuesday, down $1\frac{5}{8}$; Wednesday, down $5\frac{1}{4}$; Thursday, down $2\frac{1}{8}$; Friday, up $2\frac{1}{2}$. Find the net change in the price of IBM stock for the week. (p. 109)

The net change is figured by combining all of the daily changes for the week. (p. 109)

#12 Radio station call letters must start with W or K, like WNEW or KYRZ. (p. 189)

#13 Telephone area codes consist of 3 digits. The first digit must be chosen from 2 through 9. The second digit must be 0 or 1. The third digit cannot be 0. How many area codes are possible? (p. 189)

In a problem involving discount, the *percent of discount* is the ratio of the discount to the selling price. (p. 223)

#13 According to a teacher group, the average teacher salary for 1987-88 was $28,031. In 1870 it was $189. The 1980 salary is about how many times the 1870 salary? (p. 224)

Problems related to business often involve percents. (p. 228)

Some photocopy machines can reduce a picture to 75% or 64% of its original size. (p. 236)

Time-and-a-half means the overtime wage is $1\frac{1}{2}$ times the normal wage. (p. 238)

In 19 and 20, use these formulas relating shoe size S and foot length L in inches. for men: $s = 3L - 26$ for women: $S = 3L - 22$. (p. 265)

#24 If a discount is 10%, then the sale price is $100\% - 10\% = 90\%$ of the regular price. (p. 265)

The 4% or 8% is an example of a **markup**. (p. 272)

If an item is **discounted** x%, you pay $(100 - x)\%$ of the price (p. 273)

If an item is **marked up** or **taxed** y%, you pay $(100 + y)\%$ of the price (p. 273)

#11 Some taxicab companies allow their drivers to keep $\frac{3}{10}$ of all fares taken in. The rest goes to the company. If a driver collects F dollars in fares, write an expression for how much the company gets. (p. 274)

#28 In 1985, when IBM announced that it would no longer market the PC jr. computer, *USA Today* reported: "Omni Computer Inc. of Des Moines, Iowa, cleared out its last 12 PC jrs in one day by announcing it would cut the price 10% every hour, starting at the list of $1269, until the store closed at 5 p.m." (p. 275)

#22 The formula relating shoe size S and foot length L in women is $S = 3L - 22$. What is the approximate length of a foot of a woman who wears a size 7 shoe?

#55 If $S = 3L - 26$ relates a man's shoe size S and foot length L (in inches), find the foot length of a man with a size 11 shoe. (p. 310)

#28 From March 27, 1987 to June 29, 1987, the price of Texas intermediate crude oil went from $18.62 to $20.35 a barrel. Find the percent increase. (p. 346)

A General Motors test ramp for bulldozers goes down 0.6 feet for each foot it goes across, as illustrated at right. (p. 376)

#22 A ride in a taxicab costs a fixed number of dollars plus a constant charge per mile. (p. 397)

#25 Recall that if an item is discounted x%, you pay (100 – x)% of the original price. (p. 443)

#11 $4 \cdot 10^6$ pounds of plastic are used each year in the U.S. to produce dry cleaning bags, plastic cups, plates and the like. About how many pounds of plastic are used per person? (p. 456)

The displays in the windows are used to attract shoppers, so store owners and mall managers are interested in the areas of storefronts. (p. 486)

Calendar

#32 The number of years between historical events can be calculated by subtracting the years in which the events occurred. You will be within a year of being correct. (December 31, 1985 and January 1, 1986 are one day apart, not one year apart.) (p. 117)

There was no year 0. (p. 117)

#22 The Goldens celebrated their 50th (golden) wedding anniversary in year y. In what year were they married? (p. 121)

#27 a The year 1986 roughly corresponds to the year 2735 in the Babylonian calendar and the year 5746 in the Jewish calendar. The Norman conquest of England occurred in 1066. What year is 1066 in the Babylonian and the Jewish calendars? (p. 142)

#27 b If you graph ordered pairs (J, G), where J is the year in the Jewish calendar and G the year in the Gregorian calendar (the official one in the United States), the graph is a line. Does the line slant up or down as you go to the right? (p. 142)

Chemistry

Example 2 Water will remain ice for all temperatures less than 0° Celsius. (p. 5)

#30 When $\frac{1}{3}$ cup sugar is added to $\frac{2}{3}$ cups of coffee, the result does not fill a container that holds one cup. Why not? (p. 61)

#5 Oxygen is a liquid or solid below -183°C. Nitrogen is liquid or solid below -196°C. A scientist needs both oxygen and nitrogen in liquid or solid from. Graph the possible temperatures for obtaining both. (p. 125)

#25 The average mass of air molecules is $30 \cdot 1.66 \cdot 10^{-24}$ grams. Write this number in scientific notation. (p. 275)

Pharmacists and other chemists often face situations involving weighted averages. (p. 559)

#8 Six liters of a 10% alcohol solution are mixed with three liters of a 100% solution (that is, pure alcohol). What is the percent of alcohol in the resulting solution? (p. 560)

Consumer Education

#24 Often 15% of a restaurant bill is left for a tip. If a bill was $24.89, then what would be the tip? (p. 40)

#44 The percent discount p on an item is given by the formula $p = 100(1 - \frac{n}{g})$ where g is the original price and n is the new price. Find the percent discount on a pair of jeans whose price is reduced from $20 to $15. (p. 51)

In 46 and 47, use this information. The cost c of carpeting a room is given by $c = p(\frac{lw}{9})$, where p is the price of the carpeting per square yard and l and w are the length and width of the room in feet. (p. 52)

#30 A person usually wears shoes of the most comfortable size. Referring to the formula of Question 19, describe *all* the lengths of feet that would be best fit by a shoe of size 9. (p. 266)

Time-and-a-half means $1\frac{1}{2}$ times the regular hourly wage. (p. 297)

#55 A $\frac{3}{4}$ cup of sugar is equivalent to 12 tablespoons of sugar. How many tablespoons are there in 3 cups of sugar? (p. 258)

#12 About $1.13 \cdot 10^6$ pounds of spinach are consumed per year in the United States. How much is this per person? (p. 456)

#12 A homemade sealer to use after furniture is stained can be made by mixing one part shellac with five parts denatured alcohol. To make a pint (16 fluid ounces) of sealer, how many fluid ounces of shellac and how many of denatured alcohol are needed? (p. 539)

A ream of paper contains 500 sheets. (p. 608)

Demographics

#14 Before 1985 the average household in the United States contained more than 1.9 persons. (p. 7)

In the U.S. in 1980, of 3,612,258 children born, 1,852,616 were boys. (p. 42)

Survey results are typically reported in percent form. (p. 42)

In the example at the beginning of this lesson, the assumption might be the $\frac{1}{2}$, or 50%, of newborns are boys, but a more accurate probability would be 51.3%. (p. 42)

#1 In 1970, of 3,791,258 children born in the U.S., 1,915,378 were boys. (p. 45)

#18 Since about 51% of people born are male, why aren't 51% of all living people male? (p. 46)

#20 In 1981, a survey of 100,000 U.S. citizens showed 2284 born before 1901. What was the relative frequency of a 1981 U.S. citizen being born before 1901? (p. 49)

#51 In 1980, France had a reported population of 54,652,000. There are 9,619,000 people over the age of 60. What is the relative frequency of people 60 or under? (p. 52)

#52 One in about 86 deliveries in the U.S. results in twins. (p. 52)

#27 In 1980 the U.S. center of population was $\frac{1}{4}$ mile west of De Soto, Missouri. According to American Demographics, the center of population moves 58 feet west and 29 feet south each day. (p. 84)

#60 Draw a graph to illustrate these data (year, the U.S. population per square mile). Label the axes. (1800, 6.1), (1850, 7.9), (1900, 25.6), (1950, 50.7), (1980, 62.6). (p. 104)

Graph of population of Texas and Ohio in the years 1930-1980. (p. 105)

#30 The population of the United States is about 2.35 • 108 and the population of Los Angeles Metropolitan area is about 1.27 x 107. If a person living in the United States is selected randomly, what is the probability that the person lives in the Los Angeles area? (p. 122)

#20 The 1984 population of Illinois was about $1.14 \cdot 10^7$. The population of Chicago was about $7.10 \cdot 10^6$. A person living in Illinois is randomly selected. Approximate the probability that he or she lives in Chicago. (p. 141)

#21 In 1983, the U.S. birth rate was 15.5 babies per 1000 population. The U.S. population was 226,000,000. Use this multiplication, $15.5 \frac{\text{babies}}{1000 \text{ people}} \cdot 226{,}000{,}000$ people, to determine how many babies were born in 1983. (p. 166)

#14 On a day in the U.S., a mean of about 10,205 people each give a pint of blood. About how many days will it take to get one million pints of blood? (p. 176)

The probability that a U.S. female wears contact lenses is about $\frac{3}{50}$. (p. 191)

Picture of a street in Bangladesh. (p. 219)

#13 For each place, find the number of people per square mile. (This is called the *population density*.) (p. 219)

#13 a Bangladesh: population 85,122,000; area 55,126 sq mi. (p. 219)

#13 b Greenland: population 50,000; area 840,000 sq mi. (p. 219)

Example 3 It was reported in 1987 that 55% of the 51.5 million married couples in the U.S.A. had two incomes. Approximately how many couples had two incomes? (p. 227)

Approximately 28.3 million couples had two incomes. (p. 227)

#13 According to the census, in 1980 about 167 million people live in urban areas in the U.S. while 59.5 million live in rural areas. What percent of the U.S. population lives in a rural area? (p. 234)

rate of growth $= \frac{\text{change in population}}{\text{change in time}}$. (p. 251)

#12 The population of Arlington, Texas was about 214,000 in 1984 after a growth of 33.6% from 1980 to 1984. What was Arlington's population in 1980? (p. 274)

Example 3 The 1980 population of Columbus, OH was 565,000 people. In the 1970s it had been increasing at an average rate of 2500 people per year. In 1980 the population of Milwaukee, WI was 636,000. In the 70s it was decreasing at the rate of 8000 people a year. If these rates continued, in what year would the populations in Columbus and Milwaukee be the same? (p. 286)

#8 Alaska has a population of 480,000 which has been increasing at a rate of 6000 people a year. Delaware's population of 606,000 has been increasing at a rate of 4000 people a year. If the rates of increase do not change, in how many years will the populations be equal? (p. 287)

#33 In 1980, the population of Mississippi was 2,520,638 and was increasing at a rate of 27,400 people per year. If this rate continued, what was the population in 1985? (p. 300)

Below is a graph of the population of Manhattan Island (part of New York) every ten years from 1790 to 1980 (p. 364)

Graph with data points. (p. 364)

Rate of change in Manhattan population (people per year): 1900-1910, 48,144.9; 1910-1920, -4,743.9; 1960-1970, -15,904.8 (pp. 366-367)

Example 3 The population of the province of Ontario in Canada was 8,543,000 in 1980. If the population is increasing at a rate of 80,000 people per year, find an equation relating the population y to the year x in Ontario. (p. 388)

#6 The population of the province of Quebec in Canada was 6,398,000 in 1980. If the population is increasing at a rate of 40,000 people per year, find an equation relating the population of Quebec to the year. (p. 388)

#21 Of the 10,886,000 males aged 25-29 in 1980 how may were single? (p. 415)

Graph of percentage of USA population in prime marrying age groups remaining single 1970-1987. (p. 415)

Graph of median age for first marriage. (p. 415)

#16 Jamaica's population of 2,347,000 in 1986 was expected to grow exponentially by 1.2% each year for the next twenty years. (p. 432)

#10 In the 1980s, the growth rate of the population of the Central American country Honduras was bout 3.4% per year (a very high rate). The 1985 population of Honduras was estimated at 4,393,000. At this rate, what will the population be (to the nearest thousand) in the year 2000? (p. 436)

#12 If the growth rate of the 1980s continues, the U.S. population will double every 75 years. Since the U.S. population was 226.5 million in 1980, the number of people y (in millions) in the U.S. after x 75-year periods from 1980 is given by the formula $y = 226.5 \cdot 2^x$. (p. 441)

In 20-22, use this information. The human population P of the earth, x years from 1985, can be estimated by the formula $y = 5 \text{ billion } (1.017)^x$. (p. 451)

#19 Alaska has a population of $4.79 \cdot 10^5$ and an area of $1.48 \cdot 10^6$ square km. Find Alaska's population per square km. (p. 456)

Assume the population of the world to be 5 billion people. (p. 457)

#53 In 1987 there were about $5 \cdot 10^9$ people on earth. The land area of the earth is about $1.46 \cdot 108$ sq km. How many people are there per sq km? (p. 473)

#17 In 1981, of 3,020,000 high school graduates, 1,537,000 were women. (p. 480)

#15 In 1980 the population of Pittsburgh was about 424,000 and was decreasing by about 10,000 people a year. The 1980 population of Phoenix, Arizona was about 790,000 and was increasing by 20,000 people a year. (p. 544)

#14 South Carolina had a 1980 population of about 3,100,000 and was growing at a rate of 55,000 people each year. Louisiana had a 1980 population of about 4,200,000 and was growing at a rate of 55,000 people each year. If these rates continue, in how many years after 1980 will these two states have the same population? (p. 582)

#36 The Lorain, Ohio, area had a 1980 population of about 276,000 people and was growing at a rate of 1800 people each year. The Virginia Beach, Virginia, area had a 1980 population of about 262,000 people and was growing at a rate of 1950 people each year. If these rates continue, in how many years after 1980 will these two areas have the same population? (p. 584)

#26 Table of 1970 and 1984 populations of Dallas and San Antonio. (p. 598)

#20 In 1986, Cuba had a population of about 10.2 million living on about 44,218 square miles of land. Mexico had a population of 81.7 million living on 761,604 square miles. Which country was more densely populated? (p. 655)

There were about 1,928,000 boys and 1,833,000 girls born in the U.S. in 1985. (p. 658)

In fact, in the U.S. about 105 boys are born for every 100 girls born. But women live longer, so the population contains more women than men. (p. 658)

#35 Give the domain and range of this population function for Los Angeles, California. {(1850, 1610), (1900, 102479, (1950, 1970358), (1960, 2479015), (1970, 2811801), (1980, 2966850)}. (p. 674)

Graph of world population growth 10,000 B.C. to 1985. (p. 687)

Economics

#14 According to a survey in 1981, one out or four teenagers who worked earned more that $200 a month. Use the Equal Fractions Property to write three possibilities for the total number of teenagers surveyed and the number who earned more that $200. (p. 46)

Example 2 United States "exports" are goods made in the U.S. and sold in another country. United States "imports" are goods made in another country and sold in the U.S. "Net exports" are calculated by subtracting imports from exports. They tell whether money is flowing in or out of a country. (p. 76)

Table of United States Net Exports of Goods and Services. (p. 76)

Graph of United States Net Exports of Goods and Services. (p. 76)
There is no obvious trend from which to predict the future. (p. 76)
Graph of Number of Minutes the Average Factory Worker Had to Work to Buy Groceries Over the Years 1930 to 1977. (p. 78)
#48 The two largest milk-producing states are Wisconsin and California. Wisconsin produced 2.246 billion pounds of milk. Together these states produced 3.652 billion pounds of milk. How much milk was produced in California that year? (p. 103)
Example 2 In 1986, total medical costs in the United States were about $458,200,000,000. That year the population was about 240,500,000. What did a person spend of medical costs on the average? (p. 216)
#15 In 1986 the national debt was about 1.7 trillion dollars and was growing at a rate of about 7% per year. At this rate, find the national debt in 1994. (p. 432)
#46 In 1986, the United States had an inflation rate of about 4% per year. (p. 472)

Entertainment

#19 In 1983, the typical American teenager watched TV 15% of the time. (p. 19)
Cartoon of Dennis the Menace. (p. 35)
#19 Mary collects and trades old records. She has 40 Bing Crosby records. If she sells n of them and buys twice as many as she sells, how many will she have in all? (p. 66)
Many TV watchers seldom change channels. So TV networks often put a new show after a popular one to boost the ratings of the new show. (p. 191)
#18 For each hour of television, there are an average of $8\frac{1}{2}$ minutes of commercials. If you watch 6 hours of television in a week, compute in your head how many commercial minutes you will see. (p. 299)

Environmental Studies

#14 Foresters in the Allegheny National Forest were asked to estimate the volume of timber in the black cherry trees in the forest. They had to do it without cutting all the black cherry trees down. They did cut down 29 trees of varying sizes.They measured the diameter of each tree and the volume (in cubic feet) of wood which the tree produced. (p. 523)
Table of Black Cherry Trees (diameter and volume) in Allegheny National Forest. (p. 523)
Scattergram of Black Cherry Trees in Allegheny National Forest. (p. 523)

Games

There are six different faces which could show on the top of a die. (p. 36)
The result of tossing a die is called an outcome. There are six different outcomes: 1, 2, 3, 4, 5, 6. (p. 36)
In the situation "selecting a card from a deck" there are 52 equally-likely outcomes. (p. 36)
The probability of "picking a 5 from a deck of cards" is $\frac{4}{52}$. (p. 36)
When all outcomes of a situation like "tossing a die" have the same probability, the die is said to be *fair* or *unbiased*. (p. 37)
For example, in many games you add numbers that you roll on two dice. (p. 38)
Picture of all possible two dice possibilities. (p. 38)
#15 Matrix of heads/tails on two coins. (p. 40)
#30 When outcomes are not equally likely, the situation is called *biased*. Biased coins are sometimes called "weighted" or "two-headed" (p. 41)

#33 In Example 4 of Lesson 1-7, two fair dice were thrown once. The probability of getting a sum of 7 was calculated to be $\frac{1}{6}$. (p. 47)

In 6 and 7, imagine tossing a fair die with six outcomes {1, 2, 3, 4, 5, 6}. (p. 125)
There are 13 hearts in the deck of 52. (p. 192)
The bull's eye circle is the smaller circle on a dart board. (p. 231)
Here is a puzzle from the book *Cyclopedia of Puzzles*, written by Sam Loyd in 1914. (p. 289)
picture of World Champion chess player Gary Kasparov. (p. 624)
In Monopoly™ and many other board games, two dice are thrown and the sum of the numbers that appear is used to make a move. (p. 656)
Picture of outcomes of two dice. (p. 656)

If the dice are fair, then each of the 36 outcomes has a probability of $\frac{1}{36}$. (p. 656)

Graph of dice outcomes with probability. (p. 657)
Example 3 The total number of hands possible in the card game bridge is about 635,000,000,000. Write this number in scientific notation. (p. 685)

Geography, Geoscience

Example 3 Elevations in the state of California range from 86 meters below sea level (in Death Valley) to 4418 meters above sea level (on top of Mt. Whitney) (p. 16)
The range of elevation in California is 4504 meters. (p. 17)
#17 b Elevations in Alaska range from sea level to 6194 meters (Mt. McKinley). (p. 34)
Map of Michigan and Indiana. (p. 175)
#29 There are about 16,000 grains of sand per cubic inch, and 1728 cubic inches per cubic foot. About how many grains of sand are in a 25 cubic foot sandbox? (p. 201)
#10 The land area of the Earth is about 57,510,000 square miles and the water surface area is about 139,440,000 square miles. (p. 234)
Distance between New York and Miami is 1100 miles. (p. 243)
Map of U.S. with following cities marked: Seattle, San Francisco, Los Angeles, Denver, Houston, Chicago, Detroit, Boston, New York, and Miami. (p. 243)
Map of Earth with North Pole, Belgrade, and Equator. (p. 398)
The latitude of a place on Earth tells how far the place is from the equator. Latitudes in the Northern Hemisphere range from 0° at the equator to 90° at the North Pole. (p. 398)
Madrid, Spain, is at 40° north latitude. (p. 400)
#21 Cities are sometimes are sometimes rated on the following factors: climate, attractions (sports, museums, etc...), cost of living, and availability of good jobs. (p. 565)
Picture of the Dallas skyline. (p. 598)
Picture of Mount Rainier, near Seattle. (p. 636)
Picture of a canal in Copenhagen, Denmark. (p. 637)
Picture of Red Square, U.S.S.R. (p. 638)
One of the wondrous sights in Seattle is the peak of Mt. Rainier. (p. 661).
Example 2 From sea level in Seattle it is 58 miles to Mount Rainier. Mount Rainier towers 14,410 ft above sea level. When viewed from Seattle, how far is the peak of Mount Rainier above the horizon? (p. 661)
The solution in Example 2 ignores the curvature of the earth, which lowers the perceived height by about 2240 feet. (p. 61)
#10 From sea level in Tacoma, Washington it is 42 miles to Mount Rainier. Find the measure of the angle needed to see the peak of Mount Rainier from Tacoma. (p. 664)

History of Mathematics

About 3500 years ago Egyptians wrote the following hieroglyphics. They tell how to find the area of a rectangle with length 10 units and width 2 units. (p. 3)

Example 2 gives instances of a general pattern first given a name by Francois Servois in 1814. He used the French word "commutatif," which means "switchable". The English name is **commutative property**. (p. 58)

The associative property was given its name by the Irish mathematician Sir William Rowan Hamilton. (p. 59)

#31 Roman numerals were the most common way of writing numbers in Western Europe from about 100 B.C. to A.D. 1700. Even in this century, they have often been used to name years and on timepieces. Match each year with its corresponding Roman numeral. (p. 112)

#32 b How many years are between the death of Archimedes in 222 B.C. and the death of Julius Caesar in A.D. 43. (p. 117)

#21 In 1577, Guillaume Gosselin published a book titled *De Arte Magna*. It contained some of the first work on positive and negative numbers. It was written in Latin. Here are some rules Gosselin wrote. 1. P in P diviso quotus est P. 2. M in M diviso quotus est P. 3. M in P diviso quotus est M. 4. P in M diviso quotus est M. Translate these rules into English. (p. 214)

#24 In 1760, the French mathematician Buffon discovered the following amazing property. (p. 235)

A very famous statement in geometry, the Pythagorean Theorem, gets its name from the Greek mathematician Pythagoras, who lived about 2500 years ago. (p. 337)

This method of finding the equation of a line was developed by René Descartes in the early 1600s. (p. 392)

The idea used below was first written by the Arab mathematician al-Khowarizmi in A.D. 825 in a book entitled *Hisab al-jabr w'al muqabalah*. (p. 609)

Industrial Art, Design

#14 A machinist is making metal rods for lamps. The metal rods are to be 1.25" in diameter with an allowable error of at most .005". (The number .005" is called the *tolerance*.) Give an interval for the possible widths w of the rods. (p. 18)

You probably have played video games in which a character moves across the video screen. Programmers of games move the characters by first imagining them on a coordinate plane. (p. 80)

Meteorology

There is an old *rule of thumb* for estimating your distance from a flash of lightning. (p. 4)

#7 Shown below are the daily high temperature readings in degrees Fahrenheit for the first 27 days of January, 1979 in Sterling, Alaska. (p. 12)

#11 The temperatures at which water is steam are all those greater than 100° Celsius. (p. 13)

In 16 and 17, Fahrenheit temperature T can be approximated by counting the chirps C a cricket makes in one minute and applying the formula $T = \frac{1}{4}C + 37$. (p. 29)

Example 1 Hurricanes happen often enough in Florida that public officials must have evacuation plans ready. These plans must take into account whether people are likely to be at home, work or school. If a hurricane hits, what is the probability that it occurs on Sunday when there are the fewest people in schools and offices? (p. 36)

#61 Below are the record high temperatures T in degrees Fahrenheit in the 50 states and the District of Columbia through 1986. (p. 53)

#32 Freezing temperatures are below 32 degrees Fahrenheit. (p. 67)

#26 Here is a list of the average monthly temperatures in Los Angeles for one year: 57, 58, 59, 62, 65, 68, 73, 74, 73, 68, 63, 58. (p. 95)

Example 2 The greatest recorded difference in temperature during a single day was in Browning, Montana, in January, 1916. During one 24-hour period, the low temperature was -56°F and the high temperature was 44°F. What was the range of the temperature? (p. 114)

#4 A most unusual recorded temperature change occurred in Spearfish, SD on January 22, 1943. Over a period of two minutes the temperature rose from -4°F to 45°F. What was the range of temperature? (p. 115)

#10 The range of temperatures that have been reached in laboratories is about $8.2 \cdot 10^6$ kelvins. The minimum recorded lab temperature is $5 \cdot 10^{-8}$ kelvins. Write a subtraction equation which you could use to solve for the maximum recorded temperature M. You do not have to solve this sentence. (p. 120)

#24 In Seattle, Washington, it rained or snowed an average of 150 days per year over a 30-year period. What was the relative frequency of precipitation. (p. 225)

Example 2 A formula relating Fahrenheit and Celsius temperatures is $F = \frac{5}{9}C + 32$. What is the Celsius equivalent of normal body temperature, 98.6°F? (p. 267)

#5 What is the Celsius equivalent of a room temperature of 68°F? (p. 269)

#23 Use the formula $F = \frac{5}{9}C + 32$ to find the Celsius equivalent of 32°F? (p. 275)

#25 Refer to the graph below. Douglas, Arizona is typical of a monsoon area, an area in which a wind system causes yearly rain to be concentrated in a few months. (p. 281)

Graph of Average monthly rainfall in Douglas Arizona (in inches). (p. 281)

#19 If $F = \frac{5}{9}C + 32$, find the Celsius equivalent of a Fahrenheit temperature of 50°. (p. 307)

#56 If $F = \frac{5}{9}C + 32$, find the Celsius equivalent of a Fahrenheit temperature of 100°. (p. 310)

Consider the following official temperature readings at the Chicago lakefront on October 27, 1985: 1 am, 60°; 2 am, 60°; 3 am, 60°. (p. 319)

Biologists have found that the number of chirps crickets make per minute is related to the temperature. The relationship is very close to being linear. When crickets chirp 124 times a minute, it is 68°F. When they chirp 172 times a minute, it is 80°F. (p. 393)

The equation $y = \frac{1}{4}x + 37$ is a formula where x = chirps per minute and y = degrees Fahrenheit. (p. 394)

It is 62° when you hear 100 chirps in a minute. (p. 394)

Crickets tend not to chirp at all below 50°F. (p. 394)

The graph below shows the linear relationship between Fahrenheit and Celsius temperatures. (p. 396)

In the table below are the latitude and mean high temperatures in April, for selected cities in the Northern Hemisphere. (p. 398)

Graph of temperatures of cities in Northern Hemisphere. (p. 399)

The points on the graph suggest that the higher the latitude, the lower the temperature. (p. 399)

Graph of temperatures of cities in Northern Hemisphere with trend line included. (p. 399)

As you move 1° North in latitude, the mean April high temperature is about 1°F lower. (p. 400)

The mean high temperature in April for Madrid is actually 64°. (p. 401)

#8 a Acapulco, Mexico is at 17°N latitude. Use the equation in Example 2 to estimate its average April High temperature. (p. 401)

#8 b The actual value for Acapulco is 87°. Give a reason why the answer in part a is closer to the actual value for Acapulco than the prediction was for Mexico City. (p. 401)

Table of January Mean Low Temperature of selected cities in the Northern Hemisphere. (p. 402)

#10 g Use you equation the predict the January mean low temperature for Acapulco. (Note: the actual mean low is 70°.) (p. 402)

Newspapers or TV weather records are possible sources of high temperatures. (p. 403)

In 27 and 28, use the temperatures (in °F) shown below recorded at O'Hare Airport in Chicago starting at 10 p.m. November 21, 1985. (p. 419)

In 21 and 22, a thermometer is taken from a room temperature of 70°F to a point outside where it is 45°F. After t minutes, the thermometer reading D is approximated by $D = 24(1 - .04t)^2 + 45$. (p. 630)

Graph of Fahrenheit-Celsius temperatures. (p. 636)

Scattergram of latitude and temperature in selected cities (p. 637)

The latitudes for Copenhagen and Moscow are the same, but the temperatures are different. (p. 639)

For instance, a written rule for the Fahrenheit-Celsius function on page 637 is: To find the Celsius temperature, subtract 32 from the Fahrenheit temperature and then multiply the difference by $\frac{5}{9}$. (p. 640)

#10 Let N(t) = the number of chirps of a cricket in a minute at a temperature t° Fahrenheit. If $N(t) = \frac{1}{4}t + 37$, for what value of t is N(t) = 60? (p. 673)

Philosophy of Mathematics

For many people, formulas are clearer and easier to use than words. (p. 3)

Formulas do not necessarily work for all possible values of the variables. (p. 26)

#27 The real number π has been calculated to more than 29.36 million decimal places. Write 29.36 million as a single number without a decimal point. (p. 29)

Fraction forms often appear in situations involving rates. Rate units can be expressed using a slash "/" or horizontal bar "-". The slash or bar is read "per." (p. 163)

In mathematics, a procedure is said to be *elegant* if it is clever and simple at the same time. (p. 186)

Often ratios are written as percentages. (p. 222)

Chunking is the process of grouping some small bits of information into a single piece of information. For instance, when reading the word "store", you don't think "s, t, o, r, e". You chunk the five letters into one word. (p. 353)

Often people use chunking as a help in memorization. (p. 357)

Chunking is a problem-solving technique by which an expression is considered as a single number. (p. 358)

When faced with a multiple choice question, it is sometimes possible to arrive at the right answer by eliminating wrong ones. This is called ruling out possibilities. (p. 521)

The Quadratic Formula is one of the most famous formulas in all of mathematics. (p. 603)

The tangent function is quite a function; it combines the concepts of slope, graphing, angles, ratios, and triangles. (p. 663)

#42 If you guess on three multiple-choice questions with four choices each, the probability you will get exactly n correct is $\frac{3!}{n!(3-n)!} \cdot (\frac{1}{4})^n \cdot (\frac{3}{4})^{3-n}$. Calculate the probability you will get exactly 2 correct. (p. 674)

Physics, Optics

#13 The force you feel when riding an elevator is caused by the acceleration of the elevator acting against the pull of gravity. $f = \frac{wa}{g}$ where f = force in pounds, w = your weight in pounds, a = acceleration of elevator in feet per second per second, and g = pull of gravity, usually taken to be 32 ft per second per second (32 ft/sec/sec). (p. 28)

#28 One of the world's most famous formulas was discovered by Albert Einstein in 1905. It is $E = mc^2$. (p. 29)

#31 In chapter 1, you saw the formula for the force f you feel riding an elevator: $f = \frac{wa}{g}$ where w = your weight, a = acceleration of the elevator, and g = 32 feet per second per second (gravity). (p. 67)

#24 An air conditioning unit with a high energy efficient ratio (EER) gives more cooling with less electricity. To find the EER of a unit, divide the BTU (British Thermal Unit) number by the number of watts. The higher the EER, the more efficient the air conditioner. $EER = \frac{BTU}{watts}$. (p. 84)

#13 To balance a seesaw, weight times distance on one side must equal weight times distance on the other. (p. 176)

#17 Density D of a material is defined as the rate M/V, where M is the weight of material and V is the volume. (p. 176)

#17 b It the density of water is 62.4 pounds per cubic foot, find the weight of 10.4 cubic feet of water. (p. 176)

An *optical illusion* is something which looks one way but is actually another. (p. 313)

Most optical illusions rely on the fact that people have poor perceptions about distance and angle measure (p. 313)

#15 If an object is dropped, it takes about $\sqrt{\frac{d}{16}}$ seconds to fall d feet. If an abject is dropped from a 100-foot building, about how long does it take to hit the ground? (p. 340)

#52 Water blocks out light. (At a depth of 10 meters it is not as bright as on the surface.) Suppose 1 meter of water lets in $\frac{9}{10}$ of the light. How much light will get through x meters of water? (p. 473)

In 23 and 24 use this formula for the height h reached by a rocket after t seconds, when fired straight up from a launching pad 6 feet off the ground, $h = 6 + 96t - 16t^2$. (p. 503)

#20 The formula $d = 0.042s^2 + 1.1s$ gives the approximate number of feet d needed to stop a car on a dry concrete road (including both reaction distance and braking distance) if the speed of the car is s miles per hour. How much farther will a car travel before stopping, if it is going 65 miles per hour instead of 55 miles per hour? (p. 512)

Suppose a baton is thrown upward from ground level at a speed of 35 feet per second. It there were no pull of gravity, the distance above the ground after t seconds would be 35t, and the baton would never come down. (p. 517)

#7 A ball is thrown upward from ground level at 45 feet per second. The distance d above the ground after t seconds is $d = 45t - 16t^2$. After how many seconds will the ball hit the ground? (p. 519)

#11 A human cannonball in the circus is fired upward from the cannon at 80 feet per second. In t seconds, the cannonball will be $80t - 16t^2$ feet above where it was fired. In how many seconds will the cannonball reach a net that is the same height as the cannon? (p. 519)

#23 A tennis ball bounced up from ground level at 8 meters per second. An equation that estimates the distance d above ground (in meters) after t seconds is $d = 8t - 5t^2$. After how many seconds will the ball return to the ground? (p. 526)

#58 Recall that $d = .042s^2 + 1.1s$ gives the number of feet d needed to stop a car traveling at s mph on a concrete road. How much farther will a car travel before stopping, if it is going 50 mph instead of 30 mph? (p. 528)

#61 An orange is thrown upward from ground level at 50 feet per second. Gravity pulls the ball down $16t^2$ feet in t seconds. An equation that gives the distance d above the ground after t seconds is $d = 50t - 16t^2$. After how many seconds will the orange hit the ground? (p. 529)

#15 When a ball on the moon is thrown upward with an initial velocity of 6 meters per second, its approximate height y after t seconds is given by $y = -0.8t^2 + 6t$. (p. 607)

#17 A rock is tossed up from a cliff with upward velocity of 10 meters per second. $h = 10t - 5t^2$ relates the height h of the rock *above the cliff* after t seconds. (p. 607)

In 23-25, in a vacuum chamber on Earth an object will drop d meters in approximately t seconds, where $d = 4.9t^2$. (p. 617)

#46 When a ball is thrown into the air with initial upward velocity of 20 meters per second, its approximate height y above the ground after x seconds is given by $y = 20x - 5x^2$. When will the ball hit the ground? (p. 634)

#48 Refer the graph below. It shows the height h in feet of a ball, t seconds after it is thrown from ground level at a speed of 64 feet per second. (p. 634)

The curve is called a *sinusoidal curve*, and it has the same shape as sound waves and radio waves. (p. 667)

Example 4 the charge of an electron is .000000000048 electrostatic units. Put this number in scientific notation. (p. 685)

Politics, Law, History

Example 1 Shown below (in a dot frequency diagram) are the ages of U.S. presidents when they were first inaugurated. (p. 15)

Presidents' ages at inauguration range from 42 to 69. (p. 16)

Sets are found outside of mathematics, but with different names. (Set, element): (herd of cattle, cow), (team, player), (committee, member), (the U.S. Senate, senator), (class, student), (place setting, plate) .(p. 30)

A corps (pronounced core) was a major subdivision of the army that had 10,000 men. (p. 55)

In the decade 1880-1889, 79 men in the eleven listed corps were killed by horse-kick. (p. 55)

Table of number of deaths by horse-kick in the Prussian Army from 1880-1889 for 11 corps. (p. 55)

#31 The discussion about horse-kicks in the Prussian Army may be hard to believe, but it is true and uses real data. (p. 61)

Graph of Party Membership of United States Senators from the years 1961-1987. (p. 142)

#41 Fifty-six people signed the Declaration of Independence. Forty people signed the U.S. Constitution. Five people signed both documents. How many people signed the Constitution or the Declaration of Independence? (p. 151)

Drawing up the Declaration of Independence are (from left to right) Benjamin Franklin, Thomas Jefferson, Robert R. Livingston, John Adams, and Roger Sherman. (p. 151)

#21 Fourteen of the first forty U.S. Presidents served before the Civil War. Of the Presidents who served before that war, half were born in Virginia. Only one of the Presidents who served after that war was born in Virginia. What is the probability that a randomly chosen U.S. President served before the Civil War and was born in Virginia? (p. 203)

#27 In 1986, salaries for state governors were as high as $100,000 (New York) and as low as $35,000 (Arkansas and Maine). (p. 225)

Example 3 In 1983, only 43,200 or 2.4% of America's enlisted active duty military personnel were college graduates. How many enlisted personnel were there in all? (p. 227)

There were approximately 1,800,000 enlisted personnel. (p. 227)
#9 About 70,000 or 12.4% of America's navy personnel are officers. (p. 229)
#10 If 2.4% of America's enlisted military personnel are college graduates, what percent are *not* college graduates? (p. 229)
#18 In the U.S. army a *squad* is usually 10 enlisted men. A *platoon* is 4 squads. A *company* is 4 platoons. A *battalion* is 4 companies. A *brigade* is 3 battalions. A *division* is 3 brigades. A *corps* is 2 divisions. A *field army* is 2 corps. How many enlisted men are there in a field army? (p. 234)
#25 In the 1988 congressional election Senator Lloyd Bentsen of Texas got 3,120,348 votes out of 5,232,113 cast. (p. 246)
In 11 and 12, use the following data of total U.S. Army personnel during and after World War II. (p. 417)
In 14 and 15, use this information. School grade point averages often use the following scale: A = 4.0, B= 3.0, C = 2.0, D = 1.0, F = 0.0 . (p. 561)
Example 3 Give the domain and range of the function {(1945, 51), (1965, 117), (1985, 159)} that associates a year with the number of members of the United Nations in that year. (The United Nations was formed in 1945.) (p. 654)
#31 Here are the total numbers of votes (to the nearest million) cast for major candidates in the presidential elections since 1940. (p. 670)

Sports

Example 2 In some bowling leagues, bowlers who average under 200 can get handicaps added to their score. The handicap H of a bowler whose average is A is often found by using the formula H = .8(200 – A). (p. 26)
A set often has different properties than its elements. For example, a team in baseball can win the World Series, but a player cannot. A cow moos, but a herd cannot. The Senate can pass legislation, but a senator cannot. A plate might have a radius, a place setting does not. (p. 30)
#24 There are 12 National League (NL) baseball fields and 14 American League (AL) fields. Six NL fields are natural grass and eleven AL fields are artificial grass. What is the relative frequency of a major league ball park having natural grass? (p.61)
#25 In the eleven-year period 1977 through 1987, Edwin Moses won 122 consecutive victories, including an Olympic gold metal, in the 400 meter hurdles. What was his mean number of victories per year? (p. 73)
Example 1 In baseball, the batter hits into a playing field. The foul lines form a 90° angle. (p. 113)
In general, if all infielders can cover d degrees and the pitcher can cover P degrees, then there are 90 – 4d – P degrees left for a hard-hit ground ball to get through. (p. 114)
#28 Here is a list of National Football League scoring leaders over a 10-year period. (p. 147)
#25 A sky diver "free falls" at about 120 mi/hr which is 176 ft./sec. How many seconds does it take the diver to "free fall" 2000 ft? (p. 167)
Some students were asked to rank these famous basketball starts in order of preference: Magic Johnson, Michael Jordan, Larry Bird, Dominique Wilkins. (p. 197)
Example 2 A baseball manager is setting a batting order for his 9 starting players. How many batting orders are possible? Answer: 362,880. (p. 198)
#12 In softball, there are 10 people who can bat. In how many ways can the manager of a softball team arrange the batting order? (p. 200)
#14 A fast runner can run a half-mile in 2 minutes. What is the average rate, then in miles per *hour*? (p. 219)
In 12 and 13, use this information. In 1985, 57.7 million people in the U.S. attended at least one game of baseball, football, basketball, or ice hockey. (p. 229)

#12 Suppose 36% of all spectators attended a baseball game. How many people attended a baseball game? (p. 229)
#13 Suppose 18.8 million spectators attended a football game. What % of the people went to a football game? (p. 229)
#21 In 1987, the Los Angeles Lakers defeated the Boston Celtics in the NBA Championship series 4 games to 2. What percent of the series games were won by the Lakers? (p. 230)
#14 A world record in the 100 meter dash, set by American Calvin Smith in 1983, is 9.93 seconds. The world record in the 200 meter dash, set by Italian Pietro Mennea in 1979, is 19.72 seconds. (p. 234)
A football field has measurements given at the right. (p. 241)
Each side of a tennis court is 27 feet by 39 feet. (p. 255)
#50 In 1987, Kevin McHale from Boston had the highest field goal shooting percentage in professional basketball, 60.4%. If he attempted 1307 shots, how many did he make? (p. 257)
#28 There are 162 games in the major league baseball season. In 1987, Eric Davis of the Cincinnati Reds had 20 home runs after 55 games. If he continued at this rate, would he have broken the all-time record of 61 home runs set by Roger Maris of the New York Yankees in 1961? (p. 265)
#15 In 1988, the women's world record for the 100 m freestyle in swimming was 54.73 seconds. It had been decreasing at a rate of 0.33 seconds a year. The men's record was 49.36 seconds and had been decreasing at 0.18 seconds a year. Assume that these rates continued. (p. 288)
#22 A mast on a sailboat is strengthened by a wire (called a stay) as shown at the left. The mast is 35 ft tall. The wire is 50 ft long. How far from the base of the mast does the wire reach? (p. 356)
Graph and table of average home runs per game of the American and National League 1981-1986. (p. 380)
#19 The length of the winning toss in the men's discus throw has shown a steady increase since the first Olympics in 1896. (p. 408)
Scattergram of Olympic Champion Men's Discus Throw 1896-1984. (p. 408)
#35 The games of the 21st Modern Olympiad were in 1976. The games of the 20th Modern Olympiad were 4 years earlier. Let y by the year of the nth summer Olympic games. Give a linear equation which relates n and y. (p. 420)
#45 Olympic Swimmers get faster and faster. (p. 421)
Scattergram of times for the winners of the Women's 400-meter freestyle for the years 1924 to 1988. (p. 421)
Table of Olympic winners of the Women's 400-meter freestyle for the years 1924 to 1988. (p. 421)
Picture of Janet Evans competing in the 1988 Olympics. (p. 421)
Table of Men's and Women's winning times in the Olympic 100-meter freestyle swimming race for all Olympic years from 1912 to 1988. (p. 531)
Scattergram of Men's and Women's winning times in the Olympic 100-meter freestyle swimming race for all Olympic years from 1912 to 1988. (p. 531)
Matt Biondi, 1988 Olympic winner in men's 100-meter freestyle. (p. 531)
Equations of men's and women's 100-meter freestyle times. (p. 532)
#27 Some experts believe that even though the women's swim times are decreasing faster than the men's, it is the ratio of the times that is the key to predictions. (p. 535)
In basketball a player scores 2 points for a basket and 1 point for a free throw. (p. 577)
A parabola is the shape of the path of a basketball tossed into a hoop as shown at left, the path of a football kicked for a field goal, and the spray of water in a fountain. (p. 587)
The path of a tossed ball is a part of a parabola. (p. 599)
With a tall defender and a well-timed jump, it is quite possible to reach a height of 8 feet 9 inches to deflect or intercept the ball. (p. 599)
#25 A pitched ball in baseball does not always follow the path of a parabola. Why not? (p. 599)

A cliff diver in Acapulco, Mexico dives from a height of approximately 27 meters to the waters below "La Quebrada" ("the break in the rocks"). (p. 603)
Graph of Women's Olympic 400-meter freestyle. (p. 637)
Graph of La Quebrada cliff diver. (p. 637)
For example, there is only one Women's Olympic winning freestyle swimming time in 1972, 4.32 minutes. (p. 638)

Surveying, Measurement

#24 Each of the following words stands for an interval of time. How long is the interval? a. decade b. century c. millennium d. fortnight e. sennight f. lustrum. (p. 19)
Recall that there are 5280 feet in a mile. (p. 25)
The area is in square units because both the numbers and the units are multiplied. (p. 156)
Volume is measured in cubic units; $in.^3$ means cubic inches. (p. 158)
#33 The abbreviation mph stands for the rate unit "miles per hour." For what rate does each of these abbreviations stand? (p. 167)
For instance, 2.54 cm = 1 inch. (p. 168)
Some other useful rates equal to 1 are $\frac{5280\text{ ft}}{1\text{ mi}}$, $\frac{16\text{ ounces}}{1\text{ pound}}$, and $\frac{1\text{ min}}{60\text{ sec}}$. (p. 168)
#18 How many feet are in x miles if 5280 feet = 1 mile? (p. 171)
#20 About how many quarts are in x liters, if 1 liter = 1.06 quarts? (p. 177)
#30 One mile is 1760 yards. How many yards are in m miles? (p. 185)
#29 There are about 16,000 grains of sand per cubic inch and 1728 cubic inches per cubic foot. About how many grains of sand are in a 25 cubic foot sandbox? (p. 201)
#18 There are 2.54 cm/in. How many centimeters are there in 8 inches? (p. 203)
#43 There are 43,560 sq ft/acre. How many square feet are there in 24 acres? (p. 205)
Either way you must change 1 quart to 32 ounces. (p. 210)
The first solution gives Joe's rate in *miles per hour*. You are familiar with meaning of this rate from speed limit signs or watching the speedometer in a car. (p. 216)
#24 *Karat* is a measure of fineness used for gold and other precious materials. Pure gold is 24-karats. Gold of 18-karat fineness is 18 parts pure gold and 6 parts other metals giving 24 parts altogether. (p. 246)
With 24 hours per day for 7 days, the person would be charged for 168 hours. (p. 276)
Remember that 1 inch = 2.54 centimeters. (p. 283)
Example 2 A crate weighs 6 kg when empty. A lemon weighs about 0.2 kg. For economical shipping the crate with lemons must weigh at least 45 kg. How many lemons should be put in the crate? (p. 291)
#7 A crate weighs 10 kg when empty. A grapefruit weighs about 0.5 kg. How many grapefruit can be put in the crate and still keep the total weight under 40 kg? (p. 292)
There are 5280^3 cubic feet in a cubic mile. (p. 457)
#22 Suppose a 30-gram necklace is 14K gold. This means $\frac{14}{24}$ of the necklace is gold and the rest is other materials. How many grams of gold are in the necklace? (p. 550)
A chart of integer powers of 10 with real measurements. (p. 683)

Technology

Most scientific calculators use the same order of operations as algebra. (p. 21)
Most computer languages follow the rules for order of operations. (p. 21)
A computer program can be used to evaluate formulas. (p. 27)
If a computer is instructed to divide by zero, it won't do it! (p. 217)

Computers are most useful when jobs require speed or a lot of repetition. (p. 279)
#19 In 1986, according to the Federal Communication Commission, 7.8% of households in the U.S. were without a telephone. Of 100,000 households, how many would you expect to be without a phone? (p. 294)
#17 Calculators and computers are getting smaller and smaller. It has been estimated that the amount of information that can be stored on a silicon chip is doubling every 18 months. If today's chips have 5000 memory locations, how many might the same size chip have in 6 years? (p. 433)
Before the days of hand-held calculators (before the early 1970s), problems with large exponents could not be approached in a first-year algebra course. (p. 464)
#16 Computers often have to do millions of calculations, so anything that reduces the number of calculations saves money. (p. 493)
#21 In 1985, the cost for a large computer to process a typical data processing job was 4¢. The job took 0.4 seconds. What is the cost per second? (p. 535)
Graphs of equations are so helpful to have that there exist calculators tat will automatically display part of a graph. (p. 594)
Calculators use the idea of functions. (p. 666)
Calculators have made it possible for people to obtain values of these functions more easily than most people ever imagined. (p. 668)
Different calculators may give different answers even when the same buttons are pushed. (p. 677)
A program is a set of instructions to the computer. (p. 690)
Programs can be saved on disk. (p. 690)

Transportation

In big cities, adults may be in traffic jams every day on their way to work. (p. 25)
Here N is the estimated number of cars on the road if they are traveling at safe distances from one another. $N = \frac{20Ld}{600 + s^2}$ N = estimated number of cars, L = number of lanes of road, d = length of road (in feet), s = average speed of the cars (in miles per hour). (p. 25)
When cars go faster there should be a greater distance between them for safety. (p. 26)
One hundred years ago, horses were a major means of transportation. Their powerful kicks caused major injuries, even death. (p. 55)
#10 By air, it is 1061 miles from Miami to St. Louis and 1724 miles from St. Louis to Seattle. Use only this information to answer the questions. (p. 98)
#54 It is 346 miles from El Paso to Phoenix and 887 miles from Dallas to Phoenix. From only this information, what can you say about the distance from Dallas to El Paso? (p. 104)
This gives the formula: $\frac{\text{distance}}{\text{time}} = \text{rate}$. (p. 175)
Example 5 How long would it take to drive from Detroit to Indianapolis, a distance of 290 miles, if you travel at 55 mph? (p. 175)
#17 How much driving time is there from Chicago to St. Louis, a distance of 300 miles, if you travel at an average of 55 mph? (p. 203)
#57 How long does it take to drive from Chicago to Minneapolis, a distance of 411 miles, if a person can average 50 mph? (p. 206)
For example, if you travel at 45 miles per hour, then in 5 hours, you will travel 225 miles because $5 \text{ hours} \cdot 45 \frac{\text{miles}}{\text{hours}} = 225 \text{ miles}$. (p. 215)
#20 In 1985, 93.5 million of the 110 million workers in the U.S. commute by car. What percent is this? (p. 241)
#38 A train travels from Newark to Trenton, a distance of 48.1 miles, in 30 minutes. At what average speed, in miles per hour, does the train travel? (p. 257)

#49 In 1983, 49% of all accidental deaths occurred in motor vehicle accidents. If 91,00 people died accidentally, how many were killed in motor accidents? (p. 257)

#53 Model trains of HO-gauge are $\frac{1}{87}$ actual size (no fooling!). If a model locomotive is 30 cm long, how long is the real locomotive it models? (p. 258)

#28 Use this diagram of mileage along Interstate 10. The distance from Los Angeles to New Orleans is 1946 miles. How far is it from San Antonio to Houston? (p. 295)

Streets of many cities are laid out in a grid pattern like the coordinate plane. (p. 347)

In 18-21, the scattergram below illustrates the wingspan and length of 2-engine and 3-engine jet planes. (p. 417)

#21 In December 1986, Jeana Yeager and Dick Rutan flew the Voyager airplane nonstop around the earth without refueling. The average rate for the 25,012-mile trip was 115 mph. How many days was this flight? (p. 555)

If you know speeds going and returning on a trip, the average speed on a trip is actually a *weighted* average of the speeds used on the trip. The weights are the amount of time traveled, not distance. (p. 558)

Vocabulary

Mathematical terms are often borrowed from ordinary vocabulary. Explain the relationship between the following phrases and the statistical meaning of the underlined word. a. median strip of a highway; b. pie a la mode. (p. 13)

But the English language presents problems. The word *between* doesn't tell you whether 18 and 35 are included. (p. 15)

The Danish letter ø is also used. (p. 31)

#33 The expression "numero uno" is Spanish for "number one." Translate 1 into three other languages. (p. 172)

In 38 and 39, use the fact that an *acronym* is a name made from first letters of words or parts of words. (p. 190)

#38 Here are some famous acronyms. Tell what the letters stand for. a. NASA b. UNICEF c. CIA d. IBM e. ICBM f. AFL-CIO g. NFL h. NATO i. UNESCO (p. 190)

#46 The Greek alphabet has 24 letters. How many 3-letter monograms are possible? (p. 205)

The phrase "per capita" means "per person." (p. 220)

The word **percent** (often written as the two words per cent) comes from the Latin words *per centum*, meaning "per 100." (p. 226)

#35 How many different two-letter "words" are possible using the 12 letters of the Hawaiian alphabet? (p. 241)

Picture of the Hawaiian alphabet. (p. 241)

A picture is worth a thousand words. (p. 245)

#13 If a picture is worth a thousand words, what is the worth of 325 words? (p. 245)

#29 The statement, "driver is to car as pilot is to airplane" is called an *analogy*. (p. 246)

Series of analogies including cultural references. (p. 246)

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